Math 5660 (Numerical Analysis I) **Fall 1998**

NAME:

FINAL

2 hours long. Open books, notes, homeworks. Calculators and/or notebooks, pcs are NOT allowed. You may leave an answer in the form $2^{10}/9!$, for example. Straightforward computations should be done! Guessed answers are NOT accepted. Good luck!

(20 pt) 1) Find the Hermite cubic interpolant to $f(x) = \sqrt{x}$ at $x = 1$ and $x = 4$.

(20 pt) 2) A certain cubic spline $s(x)$ for the knot sequence $x_0 = -2$, $x_1 = 0$, $x_2 = 1$, interpolates the data $y_0 = -7$, $y_1 = 1$, $y_2 = 4$ at the knots. If $s(x) = 2x^3 + 3x^2 + 2x + 1$ on the interval $[-2, 0]$, what is the formula for $s(x)$ on the interval $[0, 1]$?

(20 pt) 3) The Hilbert matrix has elements given by:

$$a_{ij} = (1 + i + j)^{-1}, \qquad (0 \leq i, j \leq n) \ .$$

Can a Hilbert matrix have large eigenvalues? Explain your answer.

(20 pt) 4) Find the least-squares line for $y(x) = e^x$ on $(-1, 1)$ using the inner product:

$$< f, g >= \int_{-1}^{1} fg \, d\, x \, .$$

(20 pt) 5) Let x_1, x_2, and x_3 be three distinct points and y_1, y_2, and y_3 three values. Let $p(x)$ be the Lagrange interpolating polynomial of degree ≤ 1 interpolating the values y_1 and y_2 at the points x_1 and x_2, and let $q(x)$ be the Lagrange interpolating polynomial of degree ≤ 1 interpolating y_2 and y_3 at x_2 and x_3. Prove that

$$r(x) = \frac{x_3 - x}{x_3 - x_1} p(x) + \frac{x - x_1}{x_3 - x_1} q(x)$$

is the Lagrange interpolating polynomial of degree ≤ 2 interpolating y_1, y_2, and y_3 at x_1, x_2, and x_3.

(20 pt) 6) Suppose that x^* is a fixed point of $g(x)$, that $g'(x^*) = 0$, and that $|g''(x)|$ is continuous and bounded. Starting from some x_0, define a sequence x_n by $x_{n+1} = g(x_n)$. Prove that

$$|e_{n+1}| \leq M|e_n|^2,$$

where $e_n = x_n - x^*$. Give an expression for M.