Math 5660 (Numerical Analysis I) **Fall 1996**

NAME:

FINAL

2 hours long. Open books, notes, homeworks. Calculators and/or notebooks, pcs are NOT allowed. You may leave an answer in the form $2^{10}/9!$, for example. Straightforward computations should be done! Guessed answers are NOT accepted. Good luck!

(20 pt) 1) Let $f(x) = 2x^3 + 4x^2 + 10x + 9$. Find $f[x_0, x_1, x_2, x_3, x_4, x_5]$ for $x_0 = 0, x_1 = 1, x_2 = 2, x_3 = 3, x_4 = 4, x_5 = 5$.

(20 pt) 2) Consider $f(x) = e^x$ to be defined on $[-1, 1]$.

a) If we evaluate this function in 11 points equally-spaced in this interval (including the end-points) and use polynomial interpolation through these values, then estimate how large the error is for this polyomial in this interval.

b) If we evaluate this function in the roots of the Chebyshev polynomial T_{11} and interpolate, then estimate how large the error is for this new polynomial.

(20 pt) 3) Use the extended Newton divided difference algorithm to determine a polynomial that takes the following values:

$$p(-1) = 0 \quad p'(-1) = 1 \quad p(0) = 1 \quad p'(0) = 0 \quad p''(0) = 2\,.$$

(20 pt) 4) Find the least squares straight line for $f(x) = 20x^3$ on $[-1, 1]$ using the inner product

$$< f, g >= \int_{-1}^{1} fg \, d\, x \, .$$

(20 pt) 5) Consider the iterative method given by

$$x_{n+1} = \frac{1}{3}\cos x_n \quad n \geq 0$$

a) Does this method converge to a unique solution starting with $x_0 = 0$? Justify.

b) If your answer in item a) is affirmative, then estimate the error after 20 steps of this algorithm.

(20 pt) 6) Consider

$$A = \begin{pmatrix} 1 & -2 & 0 & 0 & 0 \\ -2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & 0 \\ 0 & 0 & 0 & 10 & -1 \\ 0 & 0 & 0 & -1 & 15 \end{pmatrix}$$

a) Estimate the location of the fourth smallest (or second largest) eigenvalue of A.

b) Indicate an algorithm to evaluate this eigenvalue and write a pseudo-code indicating key input data for this particular matrix.