Math 5660 (Numerical Analysis I) **Fall 1994**

NAME: ______________________________

FINAL

Open books, notes, homeworks. You may use calculators or notebooks. Guessed answers are NOT accepted. Good luck!

1) Let us consider

$$A = \begin{bmatrix} 2/3 & 1/4 \\ 4 & 1 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \tag{1}$$

for the problem of finding $\mathbf{x} \in \mathbb{R}^2$ such that

$$A\mathbf{x} = \mathbf{b} \tag{2}$$

(10 pt) a) Compute the condition number $\kappa(A)$ using the maximum norm.

Hint: The explicit formula of A^{-1} of a 2×2 matrix is given by:

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \qquad \mathit{for} \qquad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

(10 pt) b) Assume the right-hand-side $\mathbf{b}$ is perturbed by a vector $\delta\mathbf{b}$ such that $||\delta\mathbf{b}||_\infty \leq 0.02$. Find an upper-bound for $||\delta\mathbf{x}||_\infty/||\mathbf{x}||_\infty$ where $\delta\mathbf{x}$ is the corresponding perturbation in the solution $\mathbf{x}$.

(10 pt) c) For any initial guess $\mathbf{x}^{(0)}$ will the Jacobi iterative method converge? Justify using the theory in Section 5.6. (Employing the Jacobi method numerically is NOT an acceptable answer).

(10 pt) d) For any initial guess $\mathbf{x}^{(0)}$ will the Gauss-Seidel iterative method converge? Justify using the theory in Section 5.6. (The use of the Gauss-Seidel method numerically is NOT an acceptable answer).

(15 pt) e) Let us define the following iterative method

$$M\mathbf{x}^{(k+1)} = (M - A)\mathbf{x}^{(k)} + \mathbf{b} \tag{3}$$

where M is the matrix

$$M = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} \tag{4}$$

and A and $\mathbf{b}$ are given in equation (1). Will this algorithm converge to the solution of (2) for any initial guess $\mathbf{x}^{(0)}$? Justify using the theory in Section 5.6 (a numerical computation using (3) with a specific example is NOT an acceptable answer)

2) Suppose we wish to find x that solves

$$-f(x) = 0 \tag{5}$$

To fit in the framework of a one-point iteration method, one can add x to the left- and right-hand-sides of equation (5) to write

$$x = x - f(x) \tag{6}$$

Then, an iterative method to solve (5) reads

$$x_{(k+1)} = x_{(k)} - f(x_{(k)}) \quad k = 0, 1, 2, \ldots \tag{7}$$

(10 pt) a) Consider $f(x) = x^2 - (1/4)$ and the interval $J = [1/4, 3/4]$. If we select any initial guess x_0 in the interval J, will the method (7) converge to the root $\alpha = 1/2$ in J? Justify your answer using the theory for the analysis of a one-point iteration method (Taking a guess in the interval and using the method (7) to compute a few iterations IS NOT ACCEPTABLE as a general explanation on the possible convergence or not of the method (7)).

(10 pt) b) Repeat the analysis of the previous item for $f(x) = 4x^2 - 1$ in the same interval J.