6382 Mathematical Statistics, Spring 2006: Homework #5

QUESTION 1

Let X_i have a chi-squared distribution on k_i degrees of freedom, $i = 1, ..., n$; X_i's are independent.

1. What is the distribution of $Y = \sum_{i=1}^{n} X_i$?

2. Show that the distribution of $Y = a_1X_1 + a_2X_2$ where a_1, a_2 are positive constants, is not one of our familiar distributions.

3. In general, the density of $Y = \sum_{i=1}^{n} a_iX_i$, for nonnegative numbers $\{a_i\}$, is difficult to obtain. *Suppose* you want to *approximate* the distribution of Y as X_ν^2/ν for some value of ν; i.e., approximate the distribution of $\nu \cdot Y$ by a chi-squared random variable on ν degrees of freedom. Use the method of moments to suggest an estimator of ν. What problems exist with this estimator?

4. Instead of (c), use the ratio of the squared first moment to the second central moment to suggest an estimator of ν. (Hint: $Var(X_i) = 2[E(X_i)]^2/k_i$.)

5. Let $U = S_1^2/n_1$, $V = S_2^2/n_2$, where S_1^2 and S_2^2 are the sample variances from two independent Gaussian random samples of sizes n_1 and n_2, respectively. Then the statistic $W = U + V$ is typically used in the denominator of a t-like statistic when testing for common location in two samples whose population variances cannot be assumed to be equal (the Behrens-Fisher problem). Suppose that the distribution of W is *approximately* that of a X_ν^2/ν random variable. Use your answer in (d) to obtain an estimate of ν.

QUESTION 2.

The data from Vardeman and Van Valkenburg (*Technometrics* 1999, p.208) from an R&R study are as follows ($I = 2,\ J = 5,\ K = 3$):

Source	SS	df	MS	EMS
A = Parts	0.3738600	1	0.3738600	$\sigma_e^2 + 3\sigma_{ab}^2 + 15\sigma_A^2$
B = Operators	0.0006145	4	0.0001536	$\sigma_e^2 + 3\sigma_{ab}^2 + 6\sigma_B^2$
A×B = Parts×Operators	0.0001331	4	0.0000333	$\sigma_e^2 + 3\sigma_{ab}^2$
E = Error	0.0009453	20	0.0000473	σ_e^2

Generate confidence intervals for σ_A^2, σ_B^2, σ_{AB}^2, σ_e^2, $\sigma_{repro} = \sqrt{\sigma_B^2 + \sigma_{AB}^2}$, $\sigma_{R\&R} = \sqrt{\sigma_e^2 + \sigma_B^2 + \sigma_{AB}^2}$, $\sigma_{repro}^2/\sigma_{R\&R}^2$. Compare the generalized CI for σ_e^2 with those based on the ordinary chi-squared distribution.

NOTE 1: I will have only a handout describing my lecture on Welsh §4.1 (approximate confidence intervals based on asymptotic expansions) so we can begin *hypothesis testing* next week.

NOTE 2: KK's statistical computing lecture will be held on Monday, Feb 20, 3pm.

NOTE 3: I will add 2 more problems to this homework set, so you may want to start on these two problems now.