Solutions to Assignment #04 – MATH 2421

Harder/Kawai

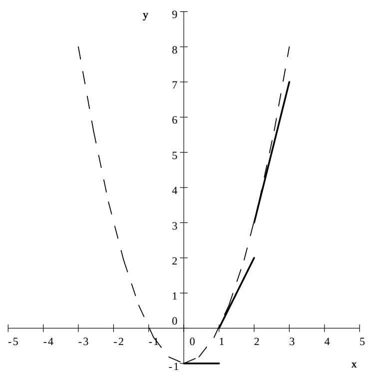
Section 9.2

(I) Complete Problems #16 & #17 on p. 770.

(#16) $\mathbf{r}(t) = \langle \cos(5t), \ \tan(t), \ 6\sin(t) \rangle$
$\mathbf{r}'(t) = \mathbf{v}(t) = \left\langle -5\sin(5t), \ \sec^2(t), \ 6\cos(t) \right\rangle.$

(#17) $\mathbf{r}(t) = \left\langle e^{t^2}, \ t^2, \ \sec(2t) \right\rangle$
$\mathbf{r}'(t) = \mathbf{v}(t) = \left\langle 2te^{t^2}, \ 2t, \ 2\tan(2t)\sec(2t) \right\rangle.$

(II) Complete Problem #20 on p. 770. Be sure to anchor the position vectors at the origin!
This must be a parabola since $x = t$ and $y = t^2 - 1 = x^2 - 1$.



The velocity function is
$\mathbf{r}'(t) = \mathbf{v}(t) = \langle 1, \ 2t \rangle.$
$\mathbf{v}(0) = \langle 1, \ 0 \rangle$
$\mathbf{v}(1) = \langle 1, \ 2 \rangle$
$\mathbf{v}(2) = \langle 1, \ 4 \rangle.$

The orientation of the parametric curve is left to right. The velocity vectors are always tangent to the parametric curve and indicate the instantaneous velocities.

(III) Complete Problem #32 on p. 771. Remember that the final answer is a vector.

We must integrate each component.

Let $u = t^2, \ du = 2t \ dt$.

$$\int_0^4 2te^{t^2} dt \quad \Rightarrow \quad \int e^u du = e^u + C$$

$$= \left[e^{t^2}\right]_0^4 = e^{16} - e^0 = e^{16} - 1.$$

Next component.

$$\int_0^4 \left(t^2 - 1\right) dt = \left[\frac{t^3}{3} - t\right]_0^4 = \frac{52}{3}.$$

Last component. Let $u = t^2 + 1, \ du = 2t \ dt \Rightarrow 4t \ dt = 2du$

$$\int_0^4 \frac{4t}{t^2+1} dt \quad \Rightarrow \quad \int 2\left(\frac{du}{u}\right) = 2\ln|u| + C$$

$$= \quad 2\left[\ln\left|t^2+1\right|\right]_0^4 = 2(\ln(17) - \ln(1)) = 2\ln(17).$$

The final answer is $\left\langle e^{16}-1,\ \frac{52}{3},\ 2\ln(17)\right\rangle$.

(IV) Complete Problem #38 on p. 771.

In each of Problems #35 & #36, show that there are NO values of t such that $\mathbf{r}(t)$ and $\mathbf{r}'(t)$ are parallel.

(#35) $\mathbf{r}(t) = \left\langle t,\ t,\ t^2-1\right\rangle$

$\mathbf{r}'(t) = \langle 1,\ 1,\ 2t\rangle$

These are parallel if and only if

$$\begin{aligned} c\langle 1,\ 1,\ 2t\rangle &= \left\langle t,\ t,\ t^2-1\right\rangle \\ c &= t \\ c &= t \\ c(2t) &= t^2-1 \end{aligned}$$

Clearly, we must have $t = c$. If we substitute this into the last equality, we have

$$2t^2 = t^2 - 1 \Rightarrow t^2 = -1.$$

This has NO real solutions. Thus, $\mathbf{r}(t)$ can never be parallel to $\mathbf{r}'(t)$.

(#36) $\mathbf{r}(t) = \left\langle t^2,\ t,\ t^2-5\right\rangle$

$\mathbf{r}'(t) = \langle 2t,\ 1,\ 2t\rangle$

These are parallel if and only if

$$\begin{aligned} c\langle 2t,\ 1,\ 2t\rangle &= \left\langle t^2,\ t,\ t^2-5\right\rangle \\ c(2t) &= t^2 \\ c &= t \\ c(2t) &= t^2-5. \end{aligned}$$

The first and third equalities both have $(2ct)$ on the left side. Thus, by substitution, we have

$$t^2 = t^2 - 5 \Rightarrow 0 = -5. \quad \text{CONTRADICTION.}$$

Thus, $\mathbf{r}(t)$ can never be parallel to $\mathbf{r}'(t)$.

(V) Complete Problem #42 on p. 771. If the velocity vector lies in the xy-plane, then something must be equal to zero...

Find all values of t such that $\mathbf{r}'(t)$ lies in the xy-plane. The z-coordinate component must be zero.

$$\mathbf{r}'(t) = \left\langle \frac{1}{2\sqrt{t+1}},\ -\sin(t),\ 4t^3-16t\right\rangle$$

Solve:

$$\begin{aligned} 4t^3-16t &= 4t\left(t^2-4\right) = 4t(t+2)(t-2) = 0 \\ t &= 0,\ -2,\ 2. \end{aligned}$$

Because of the square root in the denominator, we cannot allow $t=-2$, so we only have $t = 0,\ 2$.

Section 9.3

(VI) Find the velocity and acceleration (vector) functions for

$$\mathbf{r}(t) = \left\langle \sin(t),\ t^2\cos(t),\ 5 - t^2 \right\rangle.$$

The second component requires the Product Rule.

$$\begin{aligned} \left[t^2\cos(t)\right]' &= -t^2\sin(t) + 2t\cos(t) \\ \left[-t^2\sin(t) + 2t\cos(t)\right]' &= -t^2\cos(t) - 4t\sin(t) + 2\cos(t). \end{aligned}$$

The velocity vector function is

$$\mathbf{r}'(t) = \mathbf{v}(t) = \left\langle \cos(t),\ -t^2\sin(t) + 2t\cos(t),\ -2t \right\rangle.$$

The acceleration vector function is

$$\mathbf{r}''(t) = \mathbf{v}'(t) = \mathbf{a}(t) = \left\langle -\sin(t),\ -t^2\cos(t) - 4t\sin(t) + 2\cos(t),\ -2 \right\rangle.$$

(VII) [Similar to #14 on p. 780]

Find the position function from the given acceleration vector. This is an initial value problem.

$$\begin{aligned} \mathbf{a}(t) &= \left\langle 3t,\ e^{-t},\ \cos(t) \right\rangle \\ \mathbf{v}(0) &= \langle 1,\ 0,\ -3 \rangle \\ \mathbf{r}(0) &= \langle 0,\ 0,\ 0 \rangle \end{aligned}$$

(VIII) Complete Problem #16 on p. 780.

The formula from the text is $\mathbf{F} = m\mathbf{a}$. Thus, we need the acceleration vector.

$$\begin{aligned} \mathbf{r}(t) &= \langle 3\cos(5t),\ 3\sin(5t) \rangle \\ \mathbf{v}(t) &= \langle -15\sin(5t),\ 15\cos(5t) \rangle \\ \mathbf{a}(t) &= \langle -75\cos(5t),\ -75\sin(5t) \rangle \end{aligned}$$

We multiply this by the mass scalar $m = 10$ kg.

$$\mathbf{F}(t) = \langle -750\cos(5t),\ -750\sin(5t) \rangle \text{ Newtons.}$$

(IX) Two parts.

Complete Problem #35 on p. 780. Use English units $\left(g = -32 \text{ ft/sec}^2\right)$.

(a) Find a vector-valued function describing the position of the ball t seconds after release. We have $(x_0,\ y_0) = (0,\ 6)$ ft, and the initial speed is $v_0 = 130$ ft/sec. The initial launch angle is $\theta = 0^\circ$ since it is initially thrown horizontally.

$$\begin{aligned} \mathbf{r}(t) &= \left\langle v_0\cos(\theta)\,t + x_0,\ -16t^2 + v_0\sin(\theta)\,t + y_0 \right\rangle \\ &= \left\langle 130\cos(0)\,t,\ -16t^2 + 130\sin(0^\circ)\,t + 6 \right\rangle \\ &= \left\langle 130t,\ -16t^2 + 6 \right\rangle. \end{aligned}$$

(b) If home plate is 60 feet away, what is the altitude of the ball when it crosses above home plate?

When $x = 60$, we have

$$60 = 130t \Rightarrow t = \frac{6}{13} \text{ seconds.}$$

This is the time when the ball crosses the plate. Its height must be

$$y = -16\left(\frac{6}{13}\right)^2 + 6 = \frac{438}{160} \doteq 2.74 \text{ ft.}$$

This could easily be in the "strike zone".

(X) Complete Problem #38 on p. 781.

The details are given in #37. You must find the vector position in each case AND determine if the serve is "in" or "out".

We have $(x_0,\ y_0) = (0,\ 8)$ ft, and the initial speed is v_0 ft/sec. The initial launch angle is $\theta = 0°$ since it is initially struck horizontally.

$$\begin{aligned}\mathbf{r}(t) &= \left\langle v_0\cos(\theta)\,t + x_0,\ -16t^2 + v_0\sin(\theta)\,t + y_0\right\rangle \\ &= \left\langle v_0\cos(0)\,t,\ -16t^2 + v_0\sin(0°)\,t + 8\right\rangle \\ &= \left\langle v_0 t,\ -16t^2 + 8\right\rangle.\end{aligned}$$

(a) The initial speed is 80 ft/sec.

$$\mathbf{r}(t) = \left\langle 80t,\ -16t^2 + 8\right\rangle$$

When $x = 39$ ft, it must clear the net $(y \geq 3 \text{ ft})$.

$$\begin{aligned}39 &= 80t \Rightarrow t = \frac{39}{80} \text{ seconds} \\ y &= -16\left(\frac{39}{80}\right)^2 + 8 = \frac{1679}{400} \doteq 4.20 \text{ feet.}\end{aligned}$$

It clears the net. We now need to find out where it hits the ground $(y = 0)$.

$$y = -16t^2 + 8 = 0 \Rightarrow 16t^2 = 8 \Rightarrow t = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2} \text{ seconds.}$$

We know when it happens, regardless of the initial speed!! We substitute into the x-equation to find the associated downrange.

$$x\left(\frac{\sqrt{2}}{2}\right) = 80\left(\frac{\sqrt{2}}{2}\right) = 40\sqrt{2} \doteq 56.6 \text{ feet.}$$

The service line is 60 feet away, so the serve is "in"!

(b) The initial speed is 65 ft/sec. A slower serve may possibly not make it over the net. When $x = 39$ ft, it must clear the net $(y \geq 3 \text{ ft})$.

$$\begin{aligned} 39 &= 65t \Rightarrow t = \frac{39}{65} \text{ seconds} \\ y &= -16\left(\frac{39}{65}\right)^2 + 8 \doteq 2.24 \text{ feet.} \end{aligned}$$

The ball does not clear the net.