

**MATH 4320**  
**midterm exam**  
**Due March 20, 2008**

1. Suppose  $f_n \rightarrow f$  uniformly, and  $\forall n, g_{ni} \rightarrow f_n$  uniformly.
  - (a) Prove or find a counterexample:  $g_{nn} \rightarrow f$ .
  - (b) Now, suppose that  $\forall n, \forall i, |g_{ni} - f_n| < \frac{1}{i}$ . Prove that  $g_{nn} \rightarrow f$  uniformly.
2. Prove that if  $0 < a < b < \infty$  then

$$\int_a^b \frac{dx}{1 - e^{-x}} = b - a + \sum_{n=1}^{\infty} \frac{e^{-na} - e^{-nb}}{n}.$$

HINT: Use the geometric sum formula.

3. Let  $F : \mathfrak{R}^n \rightarrow \mathfrak{R}^m$  be continuous, and for  $A \subset \mathfrak{R}^n$  define  $F(A) = \{F(x) : x \in A\}$ .
  - (a) Prove or find a counterexample: If  $A$  is open in  $\mathfrak{R}^n$  then  $F(A)$  is open in  $\mathfrak{R}^m$ .
  - (b) Suppose that  $\forall \epsilon, \forall x, F(B_\epsilon(x)) \subset B_\epsilon(F(x))$ . Prove that  $F$  is uniformly continuous.