MATH 2423 Calc III-B, Crib Sheet for Midterm

Table of Integrals

$$\int ue^{au}\ du = \frac{au-1}{a^2}e^{au} + C$$

$$\int a^u\ du = \frac{a^u}{\ln a} + C$$

$$\int \tan u\ du = -\ln|\cos u| + C$$

$$\int \cot u\ du = \ln|\sin u| + C$$

$$\int \sec u\ du = \ln|\sec u + \tan u| + C$$

$$\int \csc u\ du = \ln|\csc u - \cot u| + C$$

$$\int \sec u \tan u\ du = \sec u + C$$

$$\int \sec^2 u\ du = \tan u + C$$

$$\int \sin^2 u\ du = \frac{u}{2} - \frac{1}{4}\sin 2u + C$$

$$\int \cos^2 u\ du = \frac{u}{2} + \frac{1}{4}\sin 2u + C$$

$$\int \tan^2 u\ du = \tan u - u + C$$

$$\int \frac{1}{\sqrt{a^2-u^2}}\ du = \sin^{-1}\frac{u}{a} + C$$

$$\int \frac{1}{a^2+u^2}\ du = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$$

$$\int \frac{1}{u^2-a^2}\ du = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + C$$

$$\int \frac{1}{a^2-u^2}\ du = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right| + C$$

$$\int \sqrt{a^2+u^2}\ du = \frac{u}{2}\sqrt{a^2+u^2} + \frac{a^2}{2}\ln\left|u+\sqrt{a^2+u^2}\right| + C$$

$$\int \frac{1}{\sqrt{a^2+u^2}}\ du = \ln\left|u+\sqrt{a^2+u^2}\right| + C$$

$$\int \sqrt{u^2-a^2}\ du = \frac{u}{2}\sqrt{u^2-a^2} - \frac{a^2}{2}\ln\left|u+\sqrt{u^2-a^2}\right| + C$$

$$\int \frac{1}{\sqrt{u^2-a^2}}\ du = \ln\left|u+\sqrt{u^2-a^2}\right| + C$$

$$\int \sqrt{a^2-u^2}\ du = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C$$

$$\int \ln u\ du = u\ln u - u + C$$

Wallis's Formulas:

1. If n is odd $(n \geq 3)$, then $\int_0^{\pi/2} \cos^n x dx = \left(\frac{2}{3}\right)\left(\frac{4}{5}\right)\left(\frac{6}{7}\right)\cdots\left(\frac{n-1}{n}\right)$.
2. If n is even $(n \geq 2)$, then $\int_0^{\pi/2} \cos^n x dx = \left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\left(\frac{5}{6}\right)\cdots\left(\frac{n-1}{n}\right)\left(\frac{\pi}{2}\right)$.

Surface area:

$$\iint_R dS = \iint_R \sqrt{1+[f_x(x,y)]^2+[f_y(x,y)]^2}\ dA.$$

Converting between rectangular and spherical coordinates:

$$\rho = \sqrt{x^2+y^2+z^2}, \quad \tan\theta = \frac{y}{x}, \quad \cos\phi = \frac{z}{\sqrt{x^2+y^2+z^2}}.$$

$$x = \rho\sin\phi\cos\theta, \quad y = \rho\sin\phi\sin\theta, \quad z = \rho\cos\phi$$

Triple integrals in spherical coordinates:

$$\iiint_Q f(x,y,z)dV = \iiint_Q f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)\rho^2\sin\phi\ d\rho\ d\phi\ d\theta.$$