Calculus Application Problem #4 **Name** ____________________________________

Walk This Way

1. The Problem

A person walks away from a motion detector at various speeds for a period of 5 seconds. We will use the calculus concepts of Rieman sums and the Fundamental Theorem of Calculus to enhance our understanding of integral calculus and the relationship between velocity and distance functions.

2. Calculating the Distance Walked

The data we have collected and graphed is a distance vs. time scatterplot of the person as he/she walks away from the motion detector. Trace on the scatterplot to determine the starting distance and the ending distance from the motion detector. Record these values below, rounded to the nearest 0.01.

Starting distance: _________________ ft. Ending distance: __________________ ft.

Subtract these distances to find the total distance walked. Record this value below.

Total distance walked: _________________ ft. (We will come back to this!)

3. The Velocity vs. Time Data and the Velocity Function

We also have the "velocity" data in the stat/data editor of our calculator. Set up and graph a scatterplot of the velocity vs. time data. This scatterplot shows the velocity of the person at any time. This is the data we want to work with! The scatterplot appears to produce a pattern that can be modeled by a quadratic function, so we will attempt to find the "best fitting" function **v(t)** in the form:

$$v(t)=at^2+bt+c.$$

To do this we will use the power of the technology and perform a quadratic regression on our calculator. (See notes, if necessary, to obtain the quadratic regression equation.) Record the equation below, with the values of **a**, **b**, and **c**, rounded to 0.01.

v(t)= ____

Enter this function into **Y1** of your calculator and graph it. Hopefully it fits the data pretty well!

4. A Few Questions

(Note: Turn off the scatterplot, and answer the following questions with the function **v(t)** that you found in part 3.)

a. What was the initial velocity of the walker? __________________

b. On what time interval was the walker slowing down? ______________________

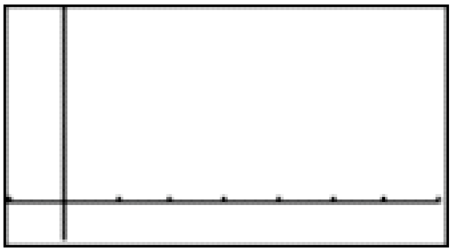
c. What was the walker's slowest speed during the 5 second period? ______________

d. What was the acceleration of the walker when t=3? _________________

5. Using the Velocity Function to Calculate the Total Distance Walked

The total distance traveled by the walker can be calculated by finding the area of the region under the velocity function and above the **t** (time) axis. We can approximate this area by dividing the region into rectangular sections and calculating the sum of the areas of the rectangles.

Sketch the graph of the velocity function below.



Our first approximation will be to divide the horizontal time axis into four equal parts. Since the total time for the data collection was 5 seconds, what is the width of each of the four rectangles?

Rectangle width = _________ secs.

Mark these values on your graph.

We will use the right endpoint of each subinterval to determine the height of each rectangle. Draw the four rectangles on your graph, and on the "top" of each rectangle write it's height (rounded to 0.01).

Finally, fill in the blanks below which show the sum of the areas of the four rectangles, and evaluate it.

Rectangle Sum = _____________ + _____________ + _____________ + _____________ = _____________

Of course, if we used more rectangles to approximate the area under the velocity function, we should get a better approximation for the total distance traveled by the walker. But, the more rectangles we use, the more work it is for us to calculate the sum of the areas of the rectangles. Unless, we can write the sum using sigma notation!

If we use 10 rectangles (of equal width) to approximate the area of the region, the width of each rectangle, which we call Δx , would be:

Rectangle width = _________ secs.

Again, if we use the right endpoint of each subinterval to determine the height of each rectangle, then the height of the first rectangle would be **v(____) =** ______ , the height of the second rectangle would be **v(_____) =** ______, the third **v(_____) =** _______, and the tenth rectangle **v(_____) =** _______**.** In general, the height of "rectangle #i" would be calculated by finding **v(_________).**

Now, write and evaluate an expression involving sigma, that could be used to calculate the area of the 10 rectangles.

Rectangle Sum = Σ ________________________ = _________________

Let's try one more approximation. If we use 40 rectangles, Δx = _____________ and the height of "rectangle #i" would be calculated by finding v(_________). Again, write and evaluate, an expression involving sigma, that could be used to calculate the area of the 40 rectangles.

Rectangle Sum = Σ ________________________ = _________________

Finally, our calculus tells us that if we use an “infinite number of rectangles”, we should get the exact area under the curve, and, therefore, the exact distance traveled by the walker. We can do this by dividing the region into **n** equal subintervals, each with width Δx = __________. The height of “rectangle #i” would be calculated by finding v(___________). Write and evaluate (using a Computer Algebra System), an expression involving sigma, that could be used to calculate the area of the **n** rectangles.

Rectangle Sum = Σ ______________________ = ___________________________

Now, if we evaluate the limit of this expression as n approaches infinity, we should have the exact area. Write and evaluate the limit expression to calculate the area of the region under the velocity function.

Area = ____________________________________ = _______________

How does this value compare to the total distance traveled by the walker in Part #2? __________________________

6. Using the Fundamental Theorem of Calculus to Calculate the Total Distance Traveled

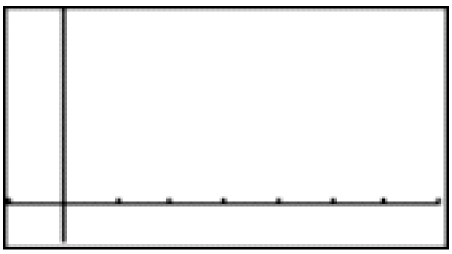
The calculus also tells us that “limits of sums” can be calculated by writing a definite integral and evaluating it using the Fundamental Theorem of Calculus. Write and evaluate a definite integral using the Fundamental Theorem of Calculus to find the area under the velocity function v(t) from t=0 to t=5. Show your work below.

7. A Few Additional Questions

a. Normally, area values are expressed in square units, such as square feet. Notice that the areas that we have computed in this activity are in units of feet (for distance), not square feet. Explain why this is so.

b. What was the average velocity of the walker during the 5 second walk? Show how you calculate this value and include units.

c. Redraw your velocity function below, and draw a horizontal line that represents the average velocity calculated above. Applying the Mean Value Theorem to this problem says that there should be at least one time during the 5 second walk that your exact rate of change (velocity) is equal to the average rate of change (average velocity). Using your graph, find the time(s) when the walker was traveling exactly the average velocity.



Time = ____________________