Calculus Application Problem #1 Name ________________________________

Heat It, then Cool It!

1. The Problem

A temperature probe is placed in a cup of hot water. It remains there for approximately 40 seconds, then it is removed from this cup and placed in a cup of coldwater for another 40 seconds. We will find an algebraic function that models the temperature recorded by the probe over the entire 80 seconds, then apply some calculus concepts to the function.

2. The General Solution (Newton's Law of Cooling/Heating)

We want to find a function **T(t)** that models the temperature **T** of the probe at any time **t**, measured in seconds. Using a property of physics, called **Newton's Law of Cooling/Heating**, the temperature in an activity such as this can be modeled by an exponential function in the form:

$$T(t) = a \cdot b^t + c \ .$$

3. The Specific Solution

First, to find an algebraic function that models our temperature vs. time data, it is clear that we need to write a piecewise function, one rule for the first (approximately) 40 seconds, and another for the last 40 (approximately) seconds. Trace on the data to find the time that separates the two rules. (t= ______ secs) To find both of these rules, we will use the property of Newton's Law of Cooling.

In order to find a model of the form $T(t) = a \cdot b^t + c$ for the first part of our data, we need to find the constants **a**, **b**, and **c**. We can find the constants **a** and **c** by "tracing" on our scatterplot.

First we will find the constant **c**. According to Newton's Law of Cooling/Heating, and the data collected, the value of **c** would be approximately ______________.

To find **a**, record the temperature when t=0. (0 , _______) Substitute this value, with the value of **c**, into our model, and solve for **a**. Show your work below.

a= _____________

To find **b**, the last constant in the model, we need another ordered pair. Trace on the data until you get to approximately 10 seconds. Record this ordered pair. (_________ , __________)
Substitute these values into the equation (with the values of **a** and **c**, and solve for the last unknown constant **b**. Show your work below.

b= _____________

T(t)= ________________

To check your work, graph your equation with your scatterplot to see how it fits the first part of the data.

The equation that fits the second part of the data is similar to the first equation. However, since we are beginning with a time other than t=0, we need to apply a "horizontal shift". Therefore, the resulting form of this function is $T(t) = a \cdot b^{t-h} + c$, where h= _______. Use this form, and the hints given for the first part of the function, and find a rule that fits this part of the data in the scatterplot. Show all of your work below. Again, graph this equation to check it.

a= _____________

b= _____________

c= _____________

T(t)= ________________

Finally, combine the results of the two parts of your rule, and write a **piecewise function** that models the temperature vs. time data.

$$T(t) = \begin{cases} \\ \\ \end{cases}$$

4. **Working with the the Temperature Function T(t).**
   Note: Do not use your scatterplot, use your function to answer these questions.

   a. Evaluate: $\lim_{t \to \infty} T(t) =$ ____________________

   b. Find the **average rate of change** of **T(t)** on the time interval [0 , 15]. Show the expression you are using.

   c. Find the **average rate of change** of **T(t)** on the time interval [50 , 65]. Show the expression you are using.

   d. Looking at the graph of the function **y=T(t)**, answer the following questions about the derivative function, **T'(t)**, which represents the **instantaneous rate of change** of the temperature at any time **t**.

   i. When, if ever, is **T'(t)** equal to zero? ______________________________

   ii. When, if ever, is **T'(t)** positive? ______________________________

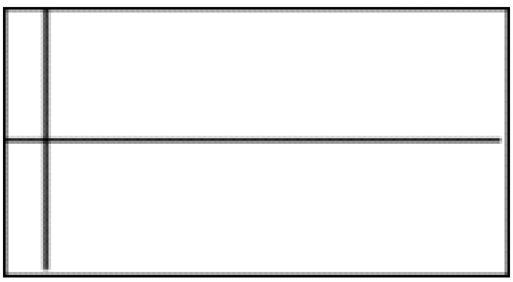
   iii. When, if ever, is **T'(t)** negative? ______________________________

   iv. When, if ever, is **T'(t)** undefined? ______________________________

   v. When, if ever, is **T'(t)** increasing in value? ______________________________

   vi. When, if ever, is **T'(t)** decreasing in value? ______________________________

   e. From your answers above, sketch a graph of **y=T'(t)** below.



f. Analytically, using derivative properties, find the rule for **T'(t)**.

Hints:

1. T'(t) will also be a piecewise function.
2. At this time, we don't have a derivative property to find the derivative of a function in the form $T(t) = a \cdot b^{t-h} + c$. This requires a property called the **Chain Rule**. However, using properties of exponents, we can write $a \cdot b^{t-h} + c$ in the form $a \cdot b^{t} + c$. This will change the value of **a**. Show this algebra below.

Find the derivative of the two parts of T(t). Show your work and write your final answer below.

$$T'(t) = \begin{cases} \\ \\ \end{cases}$$

Verify your answer by graphing it on your calculator and comparing it to the graph generated in part e above.

g. Using the function above, how fast is the temperature changing when **t=0, t=15, and t=60**? State the units with your answer.