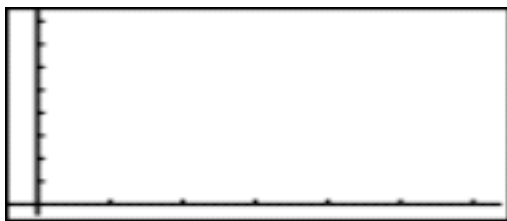


Fuel It Up!

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a differentiable function R of time t . A table of selected values of $R(t)$, for the time interval $0 \leq t \leq 90$ minutes is shown below

t	0	15	30	45	60	75	90
R(t)	20	35	63	78	73	60	56

1. On your calculator, make a **scatterplot** of the ordered pairs in the chart above. Also, show the scatterplot below and connect the points to show the continuous function $y = R(t)$. (Note: When connecting the points, assume the maximum occurs at the point (45,78)).



2. Using the data from the table, calculate an approximation for $R'(30)$ and $R'(75)$. Show the computations that lead to your answers, and indicate the units of measure.

3. The rate of fuel consumption is increasing the fastest at time $t=22.5$. What is the value of $R''(22.5)$? _____

Explain your reasoning. _____

4. If $A(x) = \int_0^x R(t) dt$, answer the following questions about $A(x)$.

a. Where is $A(x)$ concave up? _____ What is true about $R(t)$ on this interval? _____

b. Where is $A(x)$ concave down? _____ What is true about $R(t)$ on this interval? _____

c. There are no maximum or minimum values on the graph of $A(x)$. Using the graph of $y = R(t)$, explain why this is true.

5. For some value b , $0 \leq b \leq 90$ minutes, explain the meaning of $A(b) = \int_0^b R(t) dt$ in terms of fuel consumption of the plane. Indicate the units of measure. _____

6. Explain the meaning of the expression $\frac{1}{b} \cdot \int_0^b R(t) dt$ in terms of fuel consumption for the plane. Indicate the units of measure. (Hint: This was a concept covered earlier in this course!) _____

7. Using a “right” Riemann sum with the six subintervals indicated by the data in the table, write and evaluate an expression that will approximate the value of $A(90) = \int_0^{90} R(t) dt$. Draw these “approximating rectangles” on your graph of $R(t)$.

Would you guess that this approximation is “less than” or “greater than” the actual value of $A(90)$? Explain your reasoning. _____

8. Using the Trapezoidal Rule, write and evaluate an expression to approximate the value of $A(90) = \int_0^{90} R(t) dt$. Indicate the units of your answer.

9. A student proposes that the function R for the rate of consumption of the fuel for the first ninety minutes, in gallons per minutes, is given by:

$$R(t) = .4t - 20 \cos(\pi t / 45) + 40$$

Sketch a graph of $R(t)$ with your scatterplot to determine if the student was correct (on $0 \leq t \leq 90$).

10. Using the function rule for $R(t)$, find $R'(30)$, $R'(75)$, and $R''(25)$. Show your work. Tell how your answers compare with your answers to part 2 and part 3.

11. Using the function rule for $R(t)$, apply the Trapezoidal Rule with 20 subintervals. Write and evaluate an expression (involving sigma) to approximate the value of $A(90) = \int_0^{90} R(t) dt$.

12. Using the function rule for $R(t)$, write and evaluate an expression to find the exact value for the total number of gallons of gasoline that the airplane consumes over the first ninety minutes of the flight. Show your work.

13. Determine the average rate of fuel consumption for the ninety minute flight. _____

14. The **Mean Value Theorem for Integrals** states that for a continuous and positive function f on a closed interval $[a,b]$, there must exist a number c in $[a,b]$, such that:

$$f(c) = \frac{1}{b-a} \cdot \int_a^b f(x) dx$$

Find all values of c in the interval $[0,90]$, which satisfies the **Mean Value Theorem for Integrals**. Show the equation needed to be solved to find the value of c . (Note: You may find c using any method you choose. That is, you can solve the equation graphically and/or with a CAS.)