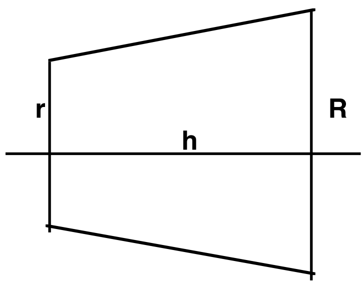
Calculus Application Problem #6 **Name** ___________________________________

Fill It Up!

I. Introduction:

A cup can be viewed as a "solid of revolution" by rotating a straight line segment about the x-axis as shown to the right. The only measurements we need to find are:



Radius of the top of the cup (R): _____________

Radius of the bottom of the cup (r): ______________

Height of the cup (h): _______________

In order to find both the volume and surface area of the cup we need to know the **equation of the line** formed by the **edge** of the cup. We can find it because we know two points on the line! The two points are:

(_______ , ________) and (________ , _________).

Use these points and find the equation of the line. Show your work.

The equation of the line that represents the edge of the cup is:

y = ______________________________

II. Volume:

1. Using calculus, find the **volume** of the cup (in cm^3). Write the integral that you are using and evaluate it using the Fundamental Theorem of Calculus. (You need to do a little algebra before finding the antiderivative.) Show all of your work.

Volume = _______________________________

(Write your answer as a decimal rounded to three decimal places. You can check your answer by evaluating the definite integral on your calculator.)

2. The volume of the cup can also be found using the geometry formula for the **volume of a cone** ($V = \frac{1}{3}\pi r^2 h$).

To make a cone out of the figure above, extend the line segment of the edge of the cone so it crosses the x-axis. We need to know the **x-intercept**. Find itusing algebra, and show your work.

x-intercept: (__________ , ___________)

Now, the volume of the cup is the volume of the ***big cone*** minus the volume of the ***top cone***. Using the geometry formula above, write the expression for the volumes of the respective cones and evaluate it.

Volume = _______________________ - ___________________________ = ____________________________

Is this close to your calculus answer? How close? _______________________________

III. Surface Area:

1. The **surface area** of a solid of revolution can be found also. Using calculus, find the surface area of the cup in cm^2. Write the integral that you are using and evaluate it, again using the Fundamental Theorem of Calculus.

Surface Area = ____

2. The surface area of the cup can also be found using the geometry formula for the **surface area of a cone** (**SA = πrL** where **L** is the **slant height** of the cone). We need to know the **two** slant heights to use the surface area formula . Find them! (Hint: The slant height is the **distance** between two points.) Show all work below.

Slant height of the ***big cone*** = ____

Slant height of the ***top cone*** = ____

Now, the surface area of the cup is the surface area of the ***big cone*** minus the surface area of the ***top cone***. Using the geometry formula above, write the two expression for the surface areas of the respective cones and evaluate it.

Surface Area = ______________________ - _________________________ = ______________________

Is this close to your calculus answer? How close? _________

IV. A Final Question:

If the "taper" of the sides of the cup remains the same, how **tall** (the height) should your cup be to be a "Big Gulp" and have a volume of 48 fluid ounces (which is approx 1420 cm^3)? Write an equation (with an integral) to solve this problem, and solve it either graphically or with the CAS on your calculator!

The height of the cup should be: _______________________________ cm.