MATH 3200 Bennethum

Review Problems for Final Exam: Laplace Transforms

These are practice problems on the Laplace trasform for the final exam. For this exam, two sides of an 8.5x11" sheet of paper will be allowed for notes (it can be on two separate sheets). No technology of any kind will be allowed. The topics covered on this review sheet include material covered in Chapter 5 Kohler and Johnson. Answers are given on the last page.

To be included with the exam: Table on forms of particular solution, Laplace transform table.

1. Take the Laplace transform of the following functions:
   (a) $1 + e^{-t}$
   (b) $e^t h(t-2)$
2. Take the inverse Laplace transform of the following functions:
   (a) $\dfrac{5}{s^2+3}$
   (b) $\dfrac{1}{s^2+3s}$
   (c) $\dfrac{e^{-s}}{s(s+1)}$
3. Solve the following initial value problem two ways: (1) Without using the Laplace transform and (2) Using the Laplace transform.
$$y'' + 4y = \sin t, \qquad y(0) = 0, \qquad y'(0) = 1$$
4. Consider the following system of linear ordinary differential equations. Find $Y_1(s)$. Do NOT find $y_1(t)$ or $y_2(t)$.
$$y_1' = y_1 + 4y_2, \qquad y_2' = -y_1 + y_2 + 3e^t$$
$$y_1(0) = 3, \qquad y_2(0) = 0$$
5. Solve the following equation for $y(t)$. The function $f(t)$ is not known.
$$y'(t) + y = f(t), \qquad y(0) = y_0.$$

1a) $\dfrac{1}{s}+\dfrac{1}{s+1}$

1b) $\dfrac{e^{2-2s}}{s-1}$

2a) $\dfrac{5\sqrt{3}}{3}\sin(\sqrt{3}t)$

2b) $\dfrac{1}{3}\left[1-e^{-3t}\right]$

2c) $h(t-1)\left[1-e^{1-t}\right]$

3) Either way you should get: $y(t)=\dfrac{1}{3}\sin t+\dfrac{1}{3}\sin 2t$

4) Note that since the equations are coupled (y_2 appears in the equation for y_1' and vice versa), we must solve it as a system of equations (there is no way to solve this by looking at just one equation). Since we are asked to find $Y_1(s)$ we must use LT. Rewriting the system in matrix form and solving we get:

$$\boldsymbol{Y}(s)=\begin{bmatrix} s-1 & -4 \\ 1 & s-1 \end{bmatrix}^{-1}\begin{bmatrix} 3 \\ \frac{3}{s-1} \end{bmatrix}$$

Inverting the matrix and multiplying out gives: $Y_1(s)=\dfrac{3s^2-6s+15}{(s-1)(s^2-2s+5)}$

5) Take the Laplace transform of both sides:

$$\begin{aligned} &sY(s)-y_0+Y=F(s), \\ &Y(s)(s+1)=F(s)+y_0 \\ &Y(s)=\frac{F(s)}{s+1}+\frac{y_0}{s+1} \\ &y(t)=f(t)*e^{-t}+y_0e^{-t} \end{aligned}$$