MATH 2421 Bennethum

Review for Test 2

Remark: This sample test is too long and heavily emphasizes setting up integrals but not evaluating them and setting up a system of equations but not solving them. For our class you will have to evaluate some integrals (this means being able to integrate using substitution and integration by parts) and be able to solve a simple system of equations.

1. Determine the location of the relative extrema for the following functions. For each extrema, determine whether it is a local max, local min, saddle point, or indeterminant. You do not need to find the z-value.
   (a) (8 points) $f(x, y) = 2x^2 + y^2 + 12x - 4y + 20$
   (b) (8 points) $f(x, y) = -x^3 + 4xy - 2y^2 + 1$
2. (8 points) Maximize $f(x, y) = 2x + 2xy + y$ subject to the constraint $2x + y = 10$. Set up the problem to the point where you have the same number of equations as unknowns. Box the equations and the list of unknowns. DO NOT SOLVE.
3. (8 points) Evaluate $\int_0^2 \int_{x/2}^1 x\,dy\,dx$.
4. (8 points) Evaluate $\int_1^\infty \int_0^\infty e^{-2x-3y}\,dy\,dx$.
5. (8 points) Set up the iterated integral needed to integrate $\dfrac{y}{1+x^2}$ over the region bounded by $y = 0$, $y = \sqrt{x}$, and $x = 4$. DO NOT EVALUATE.
6. (8 points) Change the integral to polar coordinates. DO NOT EVALUATE.
   $\int_0^a \int_0^{\sqrt{a^2-x^2}} x\,dy\,dx$

7. (8 points) Given $\int_0^1 \int_1^2 dx\, dy + \int_1^2 \int_y^2 dx\, dy$, change the order of integration to $dy\, dx$ and express it as a single iterated integral. DO NOT EVALUATE.

8. (8 points) Set up an iterated integral to find the volume under the surface $z = 4 - x^2 - 2y^2$. DO NOT EVALUATE.

9. (8 points) Set up the iterated integral needed to determine the volume of the region in the first octant that is inside the sphere $x^2 + y^2 + z^2 = 4$ and above the plane $z = 0$. *Use polar coordinates.* DO NOT EVALUATE.

10. (8 points) Set up the iterated integral needed to determine the volume of the region that is inside both the cone $z = \sqrt{x^2 + y^2}$ and the sphere $x^2+y^2+z^2 = 4$. *Use spherical coordinates.* DO NOT EVALUATE.

11. (8 points) Find the mass of a pile of snow that is in the shape of a cone $z = 2-\sqrt{x^2 + y^2}$ if the density of the snow is given by $\rho(x, y, z) = 2 - z$. EVALUATE. (Hint: Use cylindrical coordinates.)

12. (8 points) Set up the integral to determine the surface area of the paraboloid $z = 1+x^2+y^2$ that lies above the unit circle. *Use polar coordinates.* DO NOT EVALUATE.

13. (8 points) Material for the top of a rectangular box costs $\$3/ft^2$. Material for the bottom and 4 sides costs $\$5/ft^2$. What is the largest volume the box can have if the total cost of material for the box is \$96? Label your unknowns clearly. Set up this problem to the point where you have the same number of equations as unknowns. Box the equations and the unknowns. DO NOT SOLVE.

14. Determine the curl and divergence of $\boldsymbol{F} =< xyz, yz, z^2 - x^2 >$.

15. Find a vector field which is

    (a) Compressible (verify)
    (b) Incompressible (verify)
    (c) Conservative (verify)

16. Evaluate the line integral $\int_C y\,dx - 3\,dy$ where C goes from $(-1, 2)$ to $(0, 1)$.

17. Evaluate $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$ where $\boldsymbol{F} =< y, x - 2y >$ and C is the curve made of straight lines sequentially connecting $(0, 0)$, $(5, 1)$, $(3, 2)$, $(-1, 3)$, $(-2, -3)$, and $(1, -1)$.

18. Use Green's Theorem to evaluate $\int_C (\tan x - y^3)\,dx + (\sin y + x^3)\,dy$ where C is the circle $x^2 + y^2 = 2$ oriented counterclockwise. (No points for not using Green's Theorem).

1a) (-3,2) local min 1b) (0,0) saddle point; (4/3,4/3) local max
2)One possible system: $2\lambda = 2 + 2y$; $\lambda = 2x + 1$; $2x + y = 10$. Unknowns: x, y, λ. If you don't have this system, your system of equations should be satisfied identically by $x = 5/2$, $y = 5$, $\lambda = 6$.
3) 2/3
4) $\dfrac{1}{6e^2}$
5) $\displaystyle\int_0^2 \int_{y^2}^4 \frac{y}{1+x^2}\, dx\, dy$
6) $\displaystyle\int_0^{\pi/2} \int_0^a r^2 \cos\theta\, dr\, d\theta$
7) $\int_1^2 \int_0^x dy\, dx$
8) $\int_{-2}^2 \int_{-\sqrt{2-\frac{1}{2}x^2}}^{\sqrt{2-\frac{1}{2}x^2}} (4 - x^2 - 2y^2) dy\, dx$ (Remark: graph was provided on original exam)
9) $\int_0^{\pi/2} \int_0^2 \sqrt{4 - r^2}\, r\, dr\, d\theta$ (Remark: graph was provided on original exam)
10) $\int_0^{2\pi} \int_0^{\pi/4} \int_0^2 \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$ (Remark: graph was provided on original exam)
11) $\int_0^{2\pi} \int_0^2 \int_0^{2-r} (2 - z) r\, dz\, dr\, d\theta = 4\pi$ (Remark: graph was provided on original exam)
12) $\int_0^{2\pi} \int_0^1 \sqrt{1 + 4r^2}\, r\, dr\, d\theta$ (Remark: graph was provided on original exam)
13) System of equations: $yz = \lambda(8y + 10z)$, $xz = \lambda(8x + 10z)$, $xy = \lambda(10x + 10y)$, $96 = 8xy + 10xz + 10yz$, Unknowns: x, y, z, λ.
14) $\boldsymbol{\nabla} \cdot \boldsymbol{F} = yz + 3z$, $\boldsymbol{\nabla} \times \boldsymbol{F} = < y, 2x + xy, -xz >$. 15i) Any vector field such that $\boldsymbol{\nabla} \cdot \boldsymbol{F} \neq 0$ such as $\boldsymbol{F} = < x, y, z >$: $\boldsymbol{\nabla} \cdot \boldsymbol{F} = 3$.
15ii) Any vector field such that $\boldsymbol{\nabla} \cdot \boldsymbol{F} = 0$ such as $\boldsymbol{F} = < y, z, x >$. 15iii) The trick here is to begin with the potential. So for example, I'll begin with $f(x, y, z) = xyz$. Then the associated vector field is $\boldsymbol{\nabla} f = \boldsymbol{F} = < yz, xz, xy >$. To get full credit you need to give $\boldsymbol{F}$ and f.
16) 9/2.
17) -2.
18) 6π.