MATH 2421 **Review for Exam I**

1. For these problems, let $\boldsymbol{u} = 2\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$, $\boldsymbol{v} = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$, and $\boldsymbol{w} = -\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}$. There is no partial credit for the multiple choice questions, so be careful with your arithmetic.

   (a) (4 pts) $\boldsymbol{u} \cdot \boldsymbol{v} =$
   A) -1 B) 0 C) 1 D) 2 E) 3 F) 4 G) 5.

   (b) (4 pts) $\boldsymbol{u} \times \boldsymbol{w} =$
   A) $< 0, -2, 1 >$ B) $< 0, 6, -3 >$ C) $< 0, -6, 3 >$ D) $< -4, -2, 3 >$
   E) $< -4, 6, 1 >$ F) $< -4, -6, 1 >$

   (c) (4 pts) A unit vector parallel to $\boldsymbol{u}$ is:
   A) $< -2, 1, -2 >$ B) $< 6, -3, 6 >$ C) $< \frac{2}{9}, -1, 2 >$ D) $< -\frac{2}{3}, 1, -2 >$
   E) $< \frac{2}{9}, -\frac{1}{9}, \frac{2}{9} >$ F) $< -\frac{2}{3}, \frac{1}{3}, -\frac{2}{3} >$

   (d) (8 pts) Write $\boldsymbol{w}$ as the sum of two vectors; one in the direction of $\boldsymbol{v}$ and the other orthogonal to $\boldsymbol{v}$.

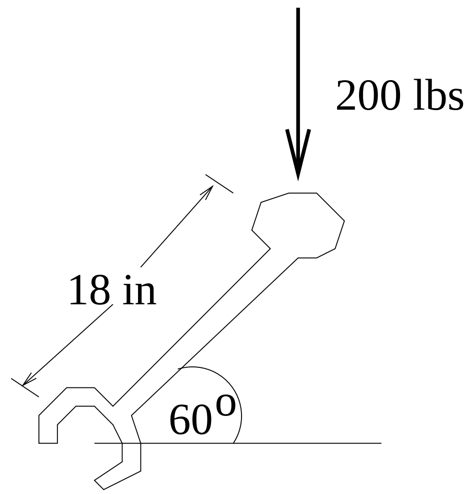
2. (8 pts) Find the parametric form of the equation of the line which passes through the point $(1, 2, 3)$ and which is orthogonal to the plane $3x + 4y - z = 4$.

3. (8 pts) The velocity and initial position of an object is given by

$$\boldsymbol{v}(t) = (3t^2 - \frac{1}{2})\boldsymbol{i} + t\boldsymbol{j}$$
$$\boldsymbol{r}(1) = 2\boldsymbol{i}$$

   Determine the position of the object when $t = 2$.

4. (8 pts) Find the torque applied by a wrench if the force of 200 pounds is applied to the end of the handle of the wrench. The handle is 18 inches long. (see picture on following page)



5. (8 pts) You want to determine the volume, V of a rectangular box. You measure its dimensions to be $l = 4$ ft, $w = 3$ ft, $h = 2$ ft, but you suspect you might have a small mistake in one of the measurements, l, w, or h. Making a mistake of 0.2 in which measurement will cause (Hint: you may want to consider differentials)

   (a) the least error, and

   (b) the greatest error in determining V.

6. (8 pts) Given

$$\boldsymbol{r}(x, y) = x^2y\boldsymbol{i} + e^{\sin(x)}\boldsymbol{j} + \pi\boldsymbol{k}$$

   find

   (a) $\dfrac{\partial \boldsymbol{r}}{\partial x}$

   (b) $\dfrac{\partial \boldsymbol{r}}{\partial y}$.

7. Consider the following curve:

$$x = 3\cos(t) \qquad y = 2\sin(t) \qquad 0 \le t \le 2\pi.$$

   (a) Eliminate t and express the curve as an equation in x and y only.

   (b) Sketch the curve, indicating the proper orientation.

   (c) Evaluate the curvature at $t = 0$ and $t = \pi/2$.

   (d) Without doing any calculations, what are the curvatures of the following curves:

i. $x = 3\cos(t)$ $\quad y = 3\sin(t)$ $\quad 0 \leq t \leq 2\pi$

ii. $x = 2\cos(t)$ $\quad y = 2\sin(t)$ $\quad 0 \leq t \leq 2\pi$

(e) Compare your answers of (b) and (d). Are you surprised? Explain.

8. (8 pts) Use the chain rule to find $\frac{\partial w}{\partial r}$ when $r = 1$ and $\theta = 2\pi$ for the function given by $w = xy + yz$ where $x = r\cos(\theta)$, $y = r\sin(\theta)$, and $z = r^2$. (No points for not using the chain rule).

9. (8 pts) The surface of a mountain is described by

$$h(x, y) = 14,000 - .01x^2 - .02y^2$$

Suppose a mountain climber is at the point $(500, 300, 9700)$ (this is a Colorado mountain!).

(a) In what direction should the climber move in order to ascend at the greatest rate?

(b) Suppose it is now winter and a skier is at the same point, wanting to go downhill as fast as possible. Which direction should the skier ski in?

10. (8 pts) Find the equation for the plane tangent to

$$h(x, y) = e^x(\sin(y) + 1)$$

at the point $(0, \pi/2, 2)$. Simplify.

11. (8 points) Mark T for True or F for False (if true, must be true for *all* vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ in three dimensions:

(a) T F If $\boldsymbol{u} \cdot \boldsymbol{v} = \boldsymbol{0}$, then either $\boldsymbol{u} = \boldsymbol{0}$ or $\boldsymbol{v} = \boldsymbol{0}$.

(b) T F The following limit,

$$\lim_{(x,y)\to(0,0)} \frac{e^{xy}}{x^2 - \cos(y)}$$

exists.

(c) T F If $\boldsymbol{T}$ and $\boldsymbol{N}$ are the unit tangent vector and principal unit normal vector, respectively, then $\boldsymbol{T} \cdot \boldsymbol{N} = 1$

(d) T F The equation of the plane $2x + 3y - z = 2$ contains the point $(2, 1, 5)$.

Answers

1a) E 1b) F 1c) F 1d) $\boldsymbol{w} = \frac{2}{3} < 1, 1, 1 > + < -\frac{5}{3}, \frac{1}{3}, \frac{4}{3} >$

2) $x = 1 + 3t$, $y = 2 + 4t$, $z = 3 - t$

3) $\frac{17}{2}\boldsymbol{i} + \frac{3}{2}\boldsymbol{j}$

4) -1800 in-lbs 5a) l, 5b) h

6a) $< 2x\sin(xy) + x^2y\cos(xy),\ e^{\sin(x)}\cos(x),\ 0 >$ 6b) $< x^3\cos(xy), 0, 0 >$

7a) $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1.$ 7b) This is an ellipse centered at the origin with the following points lying on the ellipse: $(\pm 3, 0)$, $(0, \pm 2)$. It is oriented *counter-clockwise*. 7c) $t = 0 :\ \kappa = 3/4$ $t = \pi/2 :\ \kappa = 2/9$. 7d(i)) Circle of radius 3 so curvature is 1/3. 7d(ii)) Circle of radius 2 so curvature is 1/2. 7e) I would have thought that the curvature at $t = 0$ would be close to the same as that of a circle of radius 2, maybe slightly smaller because the ellipse opens up; however $3/4 > 1/2$, so only a circle of radius smaller than 2 is tangent to the curve at $t = 0$. Likewise, I would have thought that the curvature at $t = \pi/2$ would be close to the same as that of a circle of radius 3, maybe slightly larger because the ellipse closes up near $(2, 0)$; however $2/9 < 1/3$, so a circle of radius larger than 3 is tangent to the curve at $t = \pi/2$.

8) 0 9a) $-10\boldsymbol{i} - 12\boldsymbol{j}$ 9b) $10\boldsymbol{i} + 12\boldsymbol{j}$

10) $2x - z = -2$ 11) F T F T

MATCHING PROBLEMS (Hard copy only):

5i) II 5ii) IV 5iii) III 5iv) I

6) I:D II:C III:A IV:B