

Homework 6
MATH 4830, Spring 2006
due Thurs. March 2

1. Example from class. Let X_1, \dots, X_n be a random sample of size n from the normal distribution $N(\mu, 100)$, which we have supposed is the distribution of test scores of students in a statistics course that uses a new method of teaching. We wish to decide between $H_0 : \mu = 60$ against $H_A : \mu > 60$. Let's consider a sample size of $n = 25$. We found that if we use the critical region of $\bar{x} \geq 62$, then $\alpha = P(\bar{X} \geq 62; \mu = 60) = 0.1587$ and $\beta = P(\bar{X} < 62; \mu = 65) = 0.0668$. (α and β are type I and type II error, respectively).

(a) To reduce type I error, find the c such that the critical region $\bar{x} \geq c$ is selected so that $\alpha = P(\bar{X} \geq c; \mu = 60) = 0.05$

(b) Using the c that you found in (a), find the increase in β , where $\beta = P(\bar{X} < c; \mu = 65)$.

2. A botanist is interested in comparing the growth response of dwarf pea stems to two different levels of the hormone indoleacetic acid (IAA). Using 16-day-old pea plants, she obtains 5-millimeter sections and floats these sections on solutions with different hormone concentrations to observe the effect of the hormone on the growth of the peas stem. Let X and Y be the independent growths that can be attributed to the hormone during the first 26 hours after sectioning for $(0.5)(10)^{-4}$ and 10^{-4} levels of concentration of IAA.

The data is growth in millimeters and can be read into R by:

```
> pea <- read.table("http://www-math.cudenver.edu/~bbailey/4830/pea.dat",  
header=T)
```

Make a new R object `newpea` that is a list with components: `newpea$X` and `newpea$Y` and contains the growth from the two groups. You can do this by the list command,

```
> newpea <- list(X=pea$growth[pea$group==1], Y=pea$growth[pea$group==2])
```

It should look like:

```
> newpea  
$X  
[1] 0.8 1.8 1.0 0.1 0.9 1.7 1.0 1.4 0.9 1.2 0.5  
  
$Y  
[1] 1.0 0.8 1.6 2.6 1.3 1.1 2.4 1.8 2.5 1.4 1.9 2.0 1.2
```

(a) We want test for Group 2, $H_0 : m = 1$ against $H_A : m > 1$, where m is the population median. By hand, calculate the sign-rank statistic, S . Give the Z -statistic and approximate one-sided p -value based on the normal approximation.

(b) We want test for Group 1, $H_0 : m = 1$ against $H_A : m > 1$, where m is the population median. By hand, calculate the sign-rank statistic, S . Give the Z -statistic and approximate one-sided p -value based on the normal approximation.

3. Return to Problem 2 data to perform a two-sample test.

(a) We want to test $H_0 : m_X = m_Y$ against $H_A : m_X < m_Y$, where m is the population median. By hand, calculate the rank-sum test statistic, T . (See p. 90, 1-4.) Give the Z -statistic and approximate one-sided p -value based on the normal approximation.

(b) Use the R function `wilcox.test`, by

```
> wilcox.test(newpea$X, newpea$Y, exact=F, correct=F)
```

Compare the W -statistic and p -value given by R to your answer in (a).