Game Theory 4390/5390, Fourth Homework Assignment

1. For the following game, draw the NTU and the TU feasible sets, and identify (geometrically or algebraically) the Pareto-optimal outcomes.

$$\begin{bmatrix} (0,4) & (3,2) \\ (4,0) & (2,3) \end{bmatrix}$$

2. For the following game, give a "collaborative mixed strategy" that will produce the expected outcome of $(3,2)$.

$$\begin{bmatrix} (1,4) & (1,2) \\ (4,0) & (4,1) \end{bmatrix}$$

3. Find the TU-solution of the following game.

$$\begin{bmatrix} (-3,4) & (2,-1) & (0,6) & (1,1) \\ (2,0) & (2,2) & (-3,0) & (1,-2) \\ (2,-3) & (-5,1) & (-1,-1) & (1,-3) \\ (-4,3) & (2,-5) & (1,2) & (-3,1) \end{bmatrix}$$

4. Find the NTU-solution for the following game, assuming that the threat point is "refusal to play the game."

$$\begin{bmatrix} (6,3) & (0,2) \\ (1,5) & (4,6) \end{bmatrix}$$

5. Find the equilibrium exchange rate for the following games with no fixed threat point.

$$\begin{bmatrix} (5,2) & (0,1) \\ (0,0) & (1,4) \end{bmatrix}$$

$$\begin{bmatrix} (3,2) & (0,5) \\ (2,1) & (1,0) \end{bmatrix}$$

6. Give an extensive form of the game of Tic Tac Toe where the first two moves are (1) X moves in the center square and (2) O moves in the upper left corner.

7. Give an extensive form of a simplified game of poker in which there are three card values, and three possible bets ($0 < b_1 < b_2$). In other words two players ante a dollars and then the game involves player one drawing one card from a deck and then either betting 0, or b_1 or b_2 dollars. Player two draws one card from a different deck, and either folds, calls, or raises (the only permitted raise is $b_2 - b$ where b is the amount that player one bet).

8. Consider a Cournot duopoly model with a more realistic price function so that
$$P(Q) = \frac{1}{4}Q^2 - 5Q + 26$$
when $Q = q_1 + q_2 \leq 10$ and $P(Q) = 1$ for $Q = q_1 + q_2 > 10$. Show that if the cost of production is $c = 1$, then for a fixed $q_2 = \frac{5}{2}$, the value of q_1 that maximizes the profit for firm one is $q_1 = \frac{5}{2}$. Explain how this implies that $q_1 = q_2 = \frac{5}{2}$ is a Nash Equilibrium production level.

Problems for 5390 students only:

1. Explain how a game might have an NTU feasible set with a non-linear boundary. Find the NTU solution of a game with feasible set $S = \{(x, y) : y \geq 0 \text{ and } y \leq 4 - x^2\}$, if the threat point is
   (a) $(0, 0)$
   (b) $(0, 1)$

2. Devise a simplified version of "draw" poker and give an extensive form version of the game.