MATH 2411 UNIFORM FINAL EXAM

May 5, 2007

Name: ____

Student Number: ____________________________

Circle your instructors and section number below.

B. MacMillan Y. Kuang Section #1	B. MacMillan M. Rodgers Section #2	B. MacMillan E. Vecharynski Section #3	J. Gould N. Hamoudi Section #4

Directions:

1. Complete the section above.
2. Put your name and section number on the next page of the test. You should have **9** pages of the test questions.
3. Show all work and be neat! If we can not follow your work, you will not receive any credit.
4. If you are confused about what a problem is asking, ask your instructor. You may not ask for hints or a verification on how you have completed a problem.
5. You are not allowed notes, calculators or computers.

DO NOT WRITE IN THIS SPACE

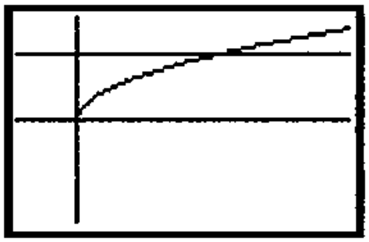
Page 1 (13 pts)	Page 2 (27 pts)	Page 3 (18 pts)
Page 4 (16 pts)	Page 5 (18 pts)	Page 6 (13 pts)
Page 7 (18 pts)	Page 8 (9 pts)	Page 9 (18 pts)

TOTAL: ____________________ (out of 150 points)

SCORE: ____________ / 150

Name ____________________________________ **Section #** ____________

1. Let **R** be the region bounded by the y-axis and the functions $y = 2$, and $y = \sqrt{x}$ as shown below.



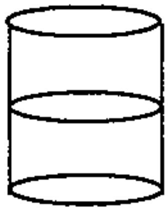
a. (5 points) Set up and evaluate the integral to find the area of **R**.

b. (4 points each) Set up, **but do not evaluate**, the integral to find the volume of the solid formed by rotating the region **R**

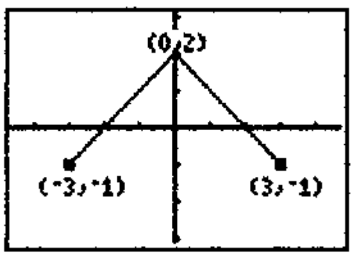
i. about the y-axis.

ii. about the line $y = 3$.

2. (5 points) The cylindrical tank shown is **half-full** of water. The height of the tank is 6 m and the diameter is 4 m. Set up, **but do not evaluate,** the integral you would use to find the amount of work necessary to pump all of the water out over the side. Note: The density of water is 1000 kg/m^3.



3. Let f be the continuous function shown below for $-3 \le x \le 3$, and let $g(x) = \int_{-1}^{x} f(t)dt$



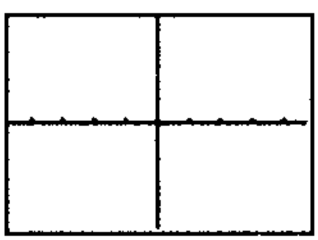
a. (2 points each) Evaluate the following:

$g(0) =$ ________ , $g(3) =$ ________ , $g'(1) =$ ________ , $g''(1) =$ ________

b. (3 points) For what values of x in (-3 , 3) is function g increasing? ________________

c. (3 points) For what values of x in (-3 , 3) is function g concave down? ________________

d. (4 points) Sketch the graph of g on $-3 \le x \le 3$



4. (4 points) If $h(x) = \int_{1}^{\ln x} \sin(t^2)dt$, then $h'(x) =$ ________________

5. (6 points each) Evaluate the following integrals. Show all work.

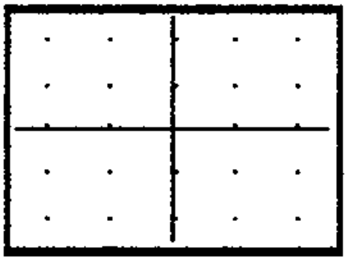
a. $\int \frac{x^2-6}{x(x-1)^2}dx$

b. $\int \frac{x^3}{\sqrt{4-x^2}}dx$

c. $\int x^2 \ln x dx$

6. Given the differential equation $\frac{dy}{dx} = -\frac{xy}{2}$,

a. (5 points) On the grid provided below, for $-2 \le x \le 2$ and $-2 \le y \le 2$, sketch a slope field at the 25 points indicated.



b. (2 points) On the slope field above, sketch the graph of the particular solution of the differential equation that contains the point (1,1).

c. (4 points) Analytically, find the particular solution to the differential equation that contains the point (1,1). Solve your answer for y.

7. (5 points) Find the particular solution to the differential equation $\frac{dy}{dx} = \frac{ye^{3x}}{y^2 - 1}$ that satisfies the initial condition $y(0) = 1$. (You do not need to solve your answer for y.)

8. (6 points) Evaluate the following improper integral. Show all work.

$$\int_0^3 \frac{x}{x^2-9}\,dx$$

SERIES TESTS
1. nth Term Test
2. Special Series Tests:
   a. Geometric Series
   b. p-series
3. Ratio Test
4. Limit Comparison Test
5. Integral Test
6. Alternating Series Test

9. (3 points) Determine if the sequence $\left\{\frac{\ln(n^2)}{n+1}\right\}$ converges or diverges. If it converges, determine the value that it converges to.

10. (5 points) Determine whether the following series converge or diverge. State the test you are using and show all work.

$$\sum_{n=1}^{\infty} n \cdot e^{-n^2}$$

11. (4 points) Determine if the following geometric series is convergent or divergent. If it is a convergent series, find its sum.

$$\frac{1}{4}-\frac{\pi}{16}+\frac{\pi^2}{64}-\frac{\pi^3}{256}+\ldots\ldots$$

12. (6 points) Determine whether the alternating series:

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n^2}}$$

is divergent or convergent. If it is convergent, determine if the series is absolutely convergent. Show all work.

13. (7 points) Determine the interval of convergence for the power series $\sum_{n=0}^{\infty} \frac{2^n (x+3)^n}{n+1}$. Show all work.

14. (8 points) Use the **definition of the Taylor series** to find the first 4 terms of the Taylor series centered at c=0 for $f(x) = e^{-3x}$. Then write the series using sigma notation.

15. (5 points each) Given the Taylor Series representation: $\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n$

Write the **first four terms** for the Taylor Series that represents each function below. Then write the series using sigma notation.

a. $x\ln\left(1+x^2\right) =$

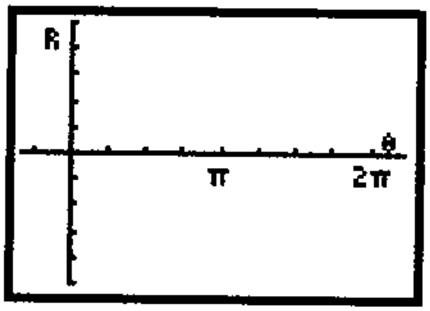
b. $\frac{1}{1+x} =$

(Hint: $\frac{d}{dx}\left(\ln(1+x)\right) = \frac{1}{1+x}$)

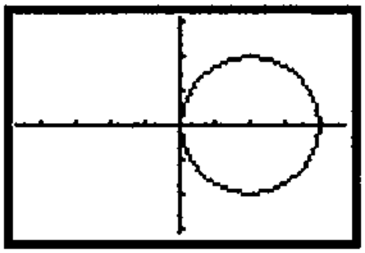
16. (3 points) Convert the point whose rectangular coordinates are (0 , -4) to polar coordinates.

(________ , ________)

17. (6 points) Given the polar equation $r = 1 + 2\sin\theta$, first sketch the graph of **r** as a function of θ in rectangular coordinates. Then use this graph to sketch the corresponding polar curve.



18. Given the polar equation $r = 4\cos\theta$, shown below



a. (6 points) Find dy/dx at $\theta = \pi/6$.

b. (6 points) Find all values of θ, $0 \le \theta \le 2\pi$, where the tangent line is horizontal. Show your work.

c. (6 points) Set up, **but do not evaluate**, the integral you would use to find the shaded area of the region shown below.