

Math 2411 Uniform Final Exam

December 6, 2008

Print your name: _____ Section: _____

Instructions:

- Put your name on this page and at the top of the next page.
- The exam is closed book and closed notes. There is a page of formulas found at the end of the exam.
- You may use a simple scientific calculator if you wish, however **no graphics calculators are allowed**.
- Answer each question in the space provided. If you need scratch paper ask for it. You may not use your own scratch paper.
- Show work where possible. Partial credit is available on most problems, but only if you show your work.
- You may not communicate with anyone other than the instructor or exam proctor during the exam. They may help to clarify the meaning of a question but they are not allowed to give you hints, verify formulas, or tell you if you have done the problem correctly.
- Be neat. If the grader cannot understand what you have written, you will not receive credit.

DO NOT WRITE BELOW THIS LINE

Page 1 (26 pts) _____

Page 2 (20 pts) _____

Page 3 (20 pts) _____

Page 4 (20 pts) _____

Page 5 (24 pts) _____

Page 6 (24 pts) _____

Page 7 (22 pts) _____

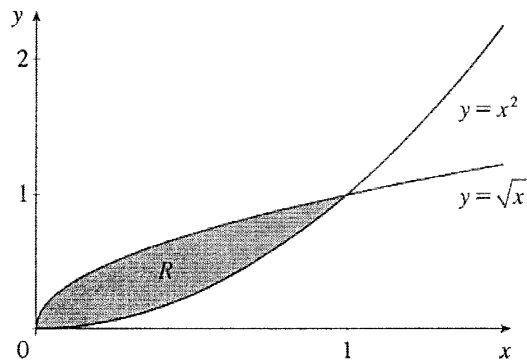
Page 8 (20 pts) _____

Page 9 (24 pts) _____

Total (200 pts) _____

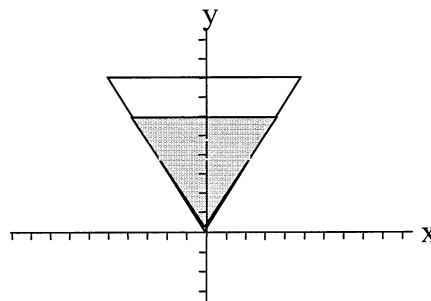
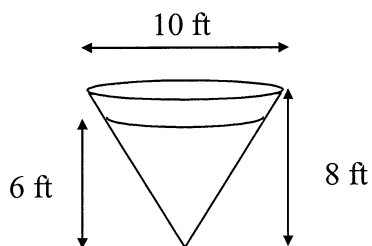
Name: _____

1. Let R be the region bounded by the y -axis and the functions $y = \sqrt{x}$, and $y = x^2$ as shown below.



- a. (10 PTS) Set up and evaluate the integral to find the area of R .
- b. (8 PTS each) Set up, but **do not evaluate**, the integral to find the volume of the solid formed by rotating the region R
- about the x -axis.
 - about the line $x = -1$.

2. (10 PTS) An open tank has the shape of a right circulate cone. The tank is 10 feet across the top and 8 feet high. The tank is initially filled with water to within 2 ft of the top of the tank. Set up, but **do not evaluate**, the integral to find how much work is done in pumping the water out over the rim of the tank? (The weight density of water is 62.4 lb/ft^3 .)



3. (10 PTS) Find the solution to the initial value problem $\frac{dy}{dx} = e^{x+y}$, $y(0) = 1$. Solve your answer for y .

4. (10 PTS each) Evaluate the following integrals. Some may be improper. Show all work.

a. $\int \frac{x-9}{x^2+3x-10} dx$

b. $\int x^2 e^x dx$

c. $\int \frac{2x}{\sqrt[3]{x^2+1}} dx$

d. $\int \frac{1}{(\sqrt{x^2+1})^3} dx$

5. (8 PTS each) Evaluate the following improper integrals. Show all work.

a. $\int_0^{\infty} 4e^{-2x} dx$

b. $\int_{-1}^1 \frac{1}{x^{2/3}} dx$

6. (8 PTS) Determine if the **sequence** $\left\{ \frac{2n^2}{3n^2 + 7} \right\}$ converges or diverges. If it converges, determine the value that it converges to.

7. (8 PTS each) Determine whether the following **series** converge or diverge. State the test you are using and show all work.

a. $1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \dots$

b. $\sum_{n=1}^{\infty} \frac{n}{n+1}$

c. $\sum_{n=1}^{\infty} \frac{1}{n^{4/3}}$

8. (10 PTS) Determine whether the alternating series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+2}$ is divergent, absolutely convergent, or conditionally convergent. Show your work.

9. (12 PTS) Determine the interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n}$. Show your work.
Don't forget to check the endpoints.

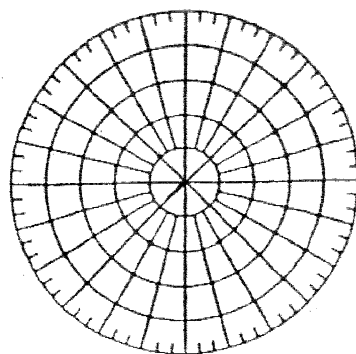
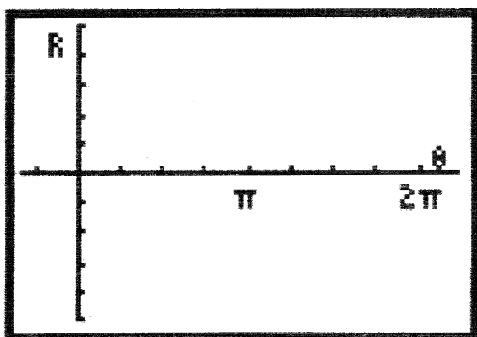
10. (12 PTS) Use the **definition of the Taylor Series** to find the first 4 terms (in simplest form) of the Taylor Series centered at $a = 1$ for $f(x) = \frac{1}{x}$. Then, write the series using sigma notation. Again, use the **definition of the Taylor Series**.

11. (8 PTS) Use one of the Six Well-Known Power Series (see last page of the exam) to derive a power series for the function $f(x) = 3xe^{3x}$. Write your answer using sigma notation.

12. (6 PTS) Convert the point whose polar coordinates are $(r, \theta) = (\sqrt{3}, \pi/6)$ to rectangular coordinates.

13. Given the polar equation $r = 1 + 2 \cos \theta$.

- a. (8 PTS) Sketch the graph of r as a function of θ in rectangular coordinates. Then use this graph to sketch the corresponding polar curve.



- b. (10 PTS) Set up, but **do not evaluate**, the integral to find the area inside the smaller loop of the graph of the limaçon from part (a) above.

Formulas

Taylor Series and Maclaurin Series

If f is a function with repeatedly differentiable derivatives, then $f(x)$ can be written as a Taylor series expansion about $x = a$ as follows:

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

If $a = 0$, the series is called a Maclaurin Series.

Six Well-Known Power Series

$$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \dots = \sum_{n=0}^{\infty} \frac{1}{n!}x^n$$

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \dots = \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n+1)!}x^{2n+1}$$

$$\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \dots = \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n)!}x^{2n}$$

$$\ln x = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \dots = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}(x-1)^n$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \dots = \sum_{n=0}^{\infty} x^n \text{ (A geometric series.)}$$

$$\tan^{-1} x = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \dots = \sum_{n=0}^{\infty} (-1)^n \frac{1}{2n+1}x^{2n+1}$$