

**FILE COPY**

MATH 1401 – UNIFORM FINAL

May 10, 2003

NAME: \_\_\_\_\_

SID#: \_\_\_\_\_

Check one	Section	Instructor	Time
	OL	Roxanne Byrne	On Line
	1	Jeff Matsuo	MW 9:00 - 10:45
	2	Jay Rothman	MW 3:00 - 4:50
	3	Dustin Stewart	TR 11:00 - 12:50
	4	Carey Jenkins	TR 1:00 - 2:50

Directions:

- Please **PRINT** your name, student ID number on both this page and the next.
- Check your section above.
- Check that you have 11 pages and 17 problems.
- No calculators or notes allowed.
- For non-multiple choice questions *show all work* and *box* your final answer.
- For multiple choice questions there will be no partial credit.
- You do not need to simplify your answers.
- You may use the back of the sheets as scratch paper, but please indicate clearly where you work each problem.

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Page 3 \_\_\_\_\_(21)      Page 10 \_\_\_\_\_(15)

Page 4 \_\_\_\_\_(16)      Page 11 \_\_\_\_\_(14)

Page 5 \_\_\_\_\_(10)

Page 6 \_\_\_\_\_(18)

Page 7 \_\_\_\_\_(14)

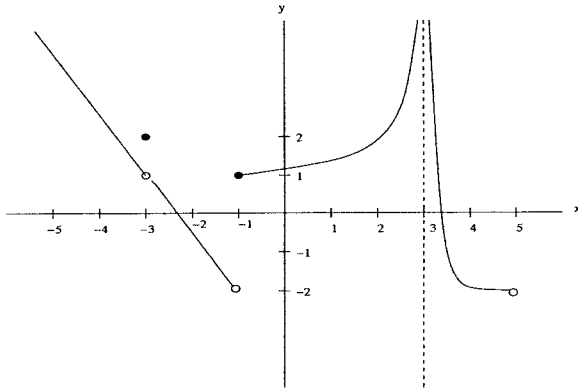
Page 8 \_\_\_\_\_(10)

**Total** \_\_\_\_\_(150)

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1. (16 pts) Given the graph of  $f$  below, circle the appropriate answer. (No partial credit.)



- (a)  $\lim_{x \rightarrow -3} f(x)$   
A) -1 B) 0 C) 1 D) 2 E)  $\infty$  F) Does not exist
- (b)  $\lim_{x \rightarrow -1} f(x)$   
A) -3 B) -2 C) -1 D) 0 E)  $\infty$  F) Does not exist
- (c)  $\lim_{x \rightarrow 5^-} f(x)$   
A) -3 B) -2 C) -1 D) 0 E)  $\infty$  F) Does not exist
- (d)  $\lim_{x \rightarrow 3} f(x)$   
A) -3 B) -2 C) -1 D) 0 E)  $\infty$  F) Does not exist
- (e)  $f$  is differentiable at  $x = 1$   
A) True B) False
- (f)  $f$  is continuous at  $x = -3$   
A) True B) False
- (g)  $f(-3) = 1$   
A) True B) False
- (h)  $f$  is continuous and differentiable on the interval  $(-3, -1)$  ( $-3 < x < -1$ ).  
A) True B) False

2. (15 pts) Evaluate the following limits.

(a) (3 pts)  $\lim_{x \rightarrow 2} \frac{x^2 + 2x - 1}{1 + 3x}$

(b) (3 pts)  $\lim_{x \rightarrow \infty} \frac{x^2 + 3x - 2}{5x^3 - 4x^2 + x}$

(c) (3 pts)  $\lim_{x \rightarrow 0^+} x \ln(x)$

(d) (3 pts)  $\lim_{x \rightarrow \infty} (x^3 - 4x)$

(e) (3 pts)  $\lim_{x \rightarrow \pi} \frac{\sin(3x)}{x - \pi}$

3. (6 pts) Find the equation of the line tangent to the curve  $f(x) = 2\sqrt{x}$  at the point  $(1, 2)$ .

4. Find  $\frac{dy}{dx}$ . Solve for  $\frac{dy}{dx}$  but it is NOT necessary to simplify your answer.

(a) (4 pts)  $y = x^2 e^x$

(b) (4 pts)  $y = \frac{x^3}{2 - \cos 3x}$

(c) (4 pts)  $y = \ln(4x^3 - 2x)$

(d) (4 pts)  $y = \ln\left(\frac{x}{2x+4}\right)$

(e) (4 pts)  $x^2y^3 + e^x = y$  (Remember to solve for  $y'$ ).

5. A curve is parametrized by  $x = t^2 - 2t$ ,  $y = t^2 - t$ .

(a) (3 pts) Where does the curve have a vertical tangent?

A) -2 B) -1 C)  $-\frac{1}{2}$  D) 0 E)  $\frac{1}{2}$  F) 1 G) 2 H) None of these.

(b) (3 pts) Where does the curve have a horizontal tangent?

A) -2 B) -1 C)  $-\frac{1}{2}$  D) 0 E)  $\frac{1}{2}$  F) 1 G) 2 H) None of these.

6. (12 pts) Let  $f(x) = 2xe^{-x}$ .

(a) Find the interval on which  $f(x)$  is increasing.

(b) (6 pts) Find the interval on which  $f(x)$  is concave upward.

7. (6 pts) Sketch the graph of a function  $f$  for which  $f(0) = 0$ ,  $f'(0) = 1$ ,  $f'(1) = 0$ , and  $f''(x) > 0$  for  $2 < x < 4$ . (There are many correct graphs, you just need to sketch one.)

8. (4 pts) The function  $f(x) = \cos(3x) + 2$  has a \_\_\_\_\_ at  $x = \pi$ .

- (a) local maxima
- (b) local minima
- (c) saddle point
- (d) none of the above

9. (4 pts) The function  $f(x) = 6x^3 - 12x + 4$  has a \_\_\_\_\_ at  $x = 0$ .

- (a) local maxima
- (b) local minima
- (c) saddle point
- (d) none of the above

10. (6 pts) Find all critical points of the function  $f(x) = e^{-x^2}$ , and classify as a relative maxima, minima, or saddle point.

11. (6 pts) Find the absolute maximum and absolute minimum of  $f(x) = 2x^3 - 6x + 2$  on the interval  $[0, 2]$ .

12. (4 pts) Find two numbers so that one is ten more than twice the other, and their product is a minimum.

13. (24 pts) Evaluate the following definite and indefinite integrals.

(a) (4 pts)  $\int_0^{\pi/2} \sin x \, dx$

(b) (4 pts)  $\int_1^4 \frac{1}{\sqrt{x}} \, dx$

(c) (4 pts)  $\int (6x^2 + e^x) \, dx$

(d) (4 pts)  $\int \frac{x^3 + x^2 + 9}{x} \, dx$

(e) (4 pts)  $\int \cos(3x) dx$  (hint: use substitution)

(f) (4 pts)  $\int \frac{x}{x^2 - 1} dx$  (hint: use substitution)

14. (4 pts) Given that  $f'(x) = \sec^2(x)$  and  $f(0) = 3$  find  $f(x)$ .

15. (3 pts) A honeybee population starts with 100 bees and increases at a rate of  $n'(t)$  bees per week. What does  $100 + \int_0^{15} n'(t) dt$  represent?

- (a) The rate of change of the number of bees after 15 weeks.
- (b) The instantaneous rate of change of the number of bees at  $t = 15$  weeks.
- (c) The number of new bees being generated at  $t = 15$  weeks.
- (d) The total number of bees after 15 weeks.
- (e) None of the above.

16. (6 pts) Two dogs finish playing and run away from each other. If one runs north at a rate of 4 feet per second and the other runs east at a rate of 3 feet per second, how is the distance between them changing after 3 seconds?

17. (8 pts) You are designing a rectangular poster to contain 50 square inches of printing (where the picture is), with a blank 4 inch margin at the top and bottom, and a blank 2 inch margin at each side. What overall dimensions of the paper will minimize the amount of total (printed and blank) paper (in square inches) used?