Uniform Final Exam – MATH 1401
Fall 2008

Name: _____

Place an 'X' by your recitation section!

( ) R01 – Mike Kawai
( ) R02 – Olson/Browning [8:00am MW]
( ) R03 – Olson/Browning [11:30am MW]
( ) R04 – Olson/Kim [12:00n TR]
( ) R05 – Olson/Kim [3:00pm TR]

Directions:

1. Please PRINT your name at the top of this page AND on the next page.
2. The first problem is on page 2 and the last problem is on page 13. You should have 20 problems total.
3. No calculators or computers.
4. Show all work on free response questions. The numbers used in the calculations are relatively small. No decimal approximations to square roots are necessary.
5. You need not show any work for multiple-choice, matching, and True/False questions.
6. Box/circle/highlight your final answers.
7. You may use the back of the sheets as scratch paper, but please indicate clearly where your work is located for each problem.

#1	/ 8	#6	/ 7	#11	/ 8	#16	/ 8
#2	/ 8	#7	/ 7	#12	/ 8	#17	/ 7
#3	/ 8	#8	/ 7	#13	/ 7	#18	/ 8
#4	/ 7	#9	/ 7	#14	/ 7	#19	/ 7
#5	/ 7	#10	/ 8	#15	/ 8	#20	/ 8
						TOTAL	150

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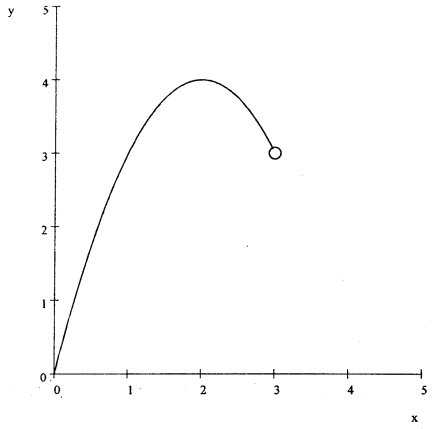
(#1) Limits.

Suppose we have the PIECEWISE function:

$$f(x) = \begin{cases} 4x - x^2, & 0 \leq x < 3 \\ x, & 3 \leq x \leq 5 \end{cases}$$

Below, I have already sketched in the parabola part in $0 \leq x < 3$.



(a) [2 *pt.*] We want you to sketch in the line segment part. Make closed dots at $(3,\ f(3))$ and $(5,\ f(5))$ and then connect them to form the line segment.

(b) [2 *pts.*] Does this limit exist? EXPLAIN! (No points awarded without an explanation!)

$\lim_{x \to 3} f(x) = ???$

(c) [2 *pts.*] Is $f(x)$ CONTINUOUS at $x = 3$? EXPLAIN! (No points awarded without an explanation!)

(d) [2 *pts.*] Does $f'(3)$ exist? EXPLAIN! (No points awarded without an explanation!)

(#2) [4 *pts. each*] L'Hôpital? Maybe. Maybe not. Evaluate the limits.

(a) $\lim_{x \to 0^+} x \ln(x) = ???$

(b) $\lim_{x \to 0^+} \frac{\cos(x)}{x} = ???$

(#3) [4 *pts. each*] Evaluate the limits.

(a) $\lim_{x \to 0} \frac{1 - \cos(x)}{x^2} = ???$

(b) $\lim_{x \to +\infty} \left(1 + \frac{1}{x^2}\right)^x = ???$

(#4) [7 *pts.*] Let $f(x) = 2x^2 - x$. Use the LIMIT DEFINITION of $f'(x)$:

$$f'(x) = \lim_{h \to 0} \frac{f(???) - f(???)}{???}$$

and show that $\left[2x^2 - x\right]' = 4x - 1$. You only receive credit for your good algebraic manipulation of this limit!

(#5) [7 *pts.*] Triple Chainburger, side order of fries.

If $h(x) = \ln(\sin(\cos(3x)))$, carefully write down $h'(x)$.

(#6) [7 *pts.*] We know that

$$f(x) = \sqrt{x}$$

is continuous on the interval $[1,\ 4]$ and differentiable on the open interval $(1,\ 4)$. then there is a guarantee via the Mean Value Theorem.

We can always find $c \in (1,\ 4)$ such that

$$f'(c) = \frac{f(???) - f(???)}{???-???} = \text{ Average rate of } \textit{something}.$$

[We hope you remember the details!] Find the correct value of c which satisfies this theorem.

(#7) [7 *pts.*] Find y' by logarithmic differentiation. DO NOT USE the Quotient Rule (= Death).

$$y = \frac{\sin^4(x)}{(x+1)^9\sqrt{x^2+1}}$$

Your final answer should be in the form:

$$y' = (\textit{something})\,(\textit{something} - \textit{something} - \textit{something}).$$

You do NOT need to combine fractions, but you must reduce your fractions when possible.

(#8) [7 *pts.*] Find y'.

$$x^3y^2 - \tan(y) + x = 1$$

(#9) Related rates.

Two men start walking from the origin. One goes north along the y-axis. At time t, he is $y(t)$ units north of the origin. (Think of it as a position function along the y-axis). Similarly, at time t, the other man goes east along x-axis and he is $x(t)$ units east of the origin.

Hint: Sketch a right triangle. The hypotenuse is $h(t)$, the distance between the two men at time t.

(a) [3 *pts.*] Find a formula for h (in terms of x and y).

(b) [4 *pts.*] Find the value of $\frac{dh}{dt}$, if $x = 3$ miles, $y = 4$ miles, $\frac{dx}{dt} = 2$ miles/hr, and $\frac{dy}{dt} = 3$ miles/hr.

(#10) Given:

$$f(x) = \frac{x^2}{x^2-1}. \quad f'(x) = \frac{-2x}{(x^2-1)^2}, \quad f''(x) = \frac{2(3x^2+1)}{(x^2-1)^3}.$$

(a) [1 *pt.*] The graph of f has one HORIZONTAL asymptote. Name it:
$y =$

(b) [1 *pt.*] Name ALL the VERTICAL asymptotes of f. [Hint: Those will all be critical numbers!]

(c) [4 *pts.*] Find the intervals where f is increasing. Find the intervals where f is decreasing. You may use the sign chart (or not). [You may or may not need all the rows.]

INTERVALS → FACTORS ↓				
f'				
Increasing? Decreasing?				

(d) [2 *pts.*] Find all intervals where the function is:
[Hint: $(3x^2+1)$ is always positive.]

Concave UP:

Concave DOWN:

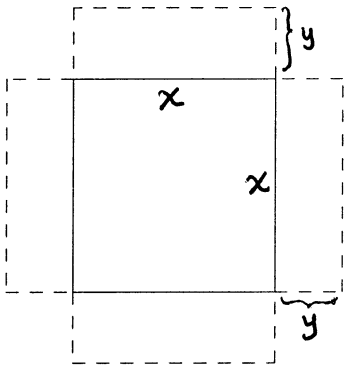
(#11) A box with an open top (no top) and a SQUARE base is to be constructed by taping up the sides of this cut-out pattern.

The surface area is equal to the area of the square plus the four other rectangles.

Suppose the volume

$$\begin{aligned} V &= (area\ of\ base) * (height) \\ 4 &= (???) * y \Rightarrow y = ??? \end{aligned}$$

is exactly 4 cubic feet.



(a) [3 *pts.*] Find the surface area in terms of x.

$A(x) = ???$

(b) [5 *pts.*] Find the only POSITIVE value of x which minimizes the surface area, $A(x)$.

(#12) [4 *pts. each*] Find $f'(x)$.

(a) $f(x) = \sin^{-1}\left(3x^4 - 5\right)$.

(b) $f(x) = \left(\sqrt{x^5}\right)\tan(x)$

(#13) [7 *pts.*] Find ALL the critical numbers ($f'(x) = 0$) in $0 \leq x < 2\pi$.
[Not knowing the double angle formula = much sadness. Factor!]

$$f(x) = -\frac{\cos(2x)}{2} + \sin(x)$$

(#14) [7 *pts.*] Find $h'(x)$.

$$h(x) = \frac{x^3}{2 + \sec(x)}$$

(#15) Suppose we have the curve $y = \ln(x)$.

(a) [5 *pts.*] Find the equation of the tangent line at the point $(1,\ 0)$.
[Point-slope form is preferred.]

(b) [3 *pts.*] Since this (above) is the linear approximation of the $\ln(x)$ function near $x = 1$, give your best decimal (linear) approximation of

$$\ln(1.002),$$

to three decimal places, since you don't have a calculator.

(#16) [4 *pts. each*] Assume that $y = f(x)$ is a POLYNOMIAL. In each group of choices, circle the correct choice which fits the text.

If $f(x)$ has exactly two zeros at $x = a$ and $x = b$, then we are guaranteed

(**exactly one / at least one / no**) point(s) in between $(a < c < b)$ where $f'(c) = 0$ by Rolle's Theorem.

Whenever the derivative graph $y = f'(x)$ CHANGES DIRECTION, we have

(**a horizontal tangent line / a vertical tangent line / an inflection point**) on the original graph of $y = f(x)$.

(#17) Suppose $y(t)$ represents the position of a particle along the y-axis. Hence, the particle is moving up and down along the y-axis.

If the velocity function is

$$\frac{dy}{dt} = \sin(t) + 1, \text{ with } y(0) = \frac{1}{2},$$

then we know that the particle is located at $y = 1/2$ when $t = 0$.

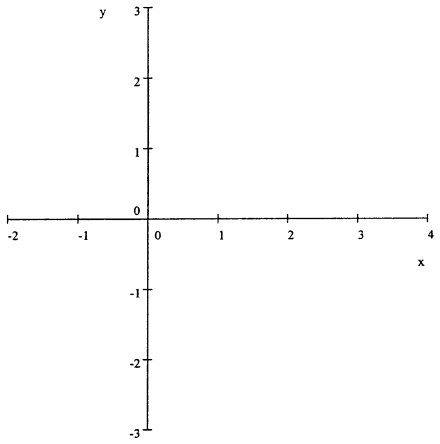
(a) [5 *pts.*] Find $y(t)$. [You have enough information to find C now!]

(b) [2 *pts.*] Find the acceleration function, in terms of t.

(#18) Positive and negative area.

(a) [2 *pts.*] Sketch the line $y = 1 - \frac{x}{2}$ from $x = -2$ to $x = 4$ on the axes below.

[Hint: Part of the line is below the x-axis!]



(b) [2 *pts.*] From geometry, how much trapped area (between the line and the x-axis) is POSITIVE?

How much is NEGATIVE?

(c) [4 *pts.*] Use the Fundamental Theorem to evaluate:

$$\int_{-2}^{4} \left(1 - \frac{x}{2}\right) dx = ???$$

Does this answer make sense, according to your answers from parts (a) & (b)?

(#19) [7 *pts.*] Evaluate the definite integral. It represents trapped area!

$$\int_1^4 \left(x^2 + \sqrt{x}\right) dx = ???$$

(#20) [4 *pts. each*] Find the antiderivatives $(+C)$.

Hint: u should be able to do this, shouldn't u?

(a) $\displaystyle\int \frac{3x^2+3}{\sqrt{x^3+3x}} dx = ???$

(b) $\displaystyle\int \frac{e^x}{1+e^{2x}} dx = ???$ Tricky.