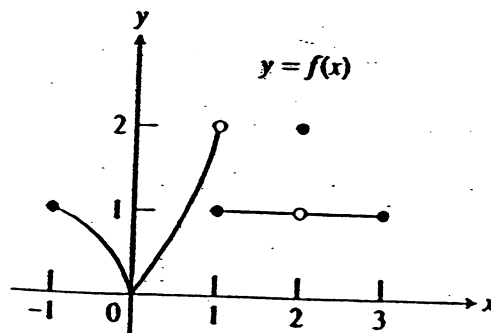


Name _____

1. (2 points each) The complete graph of $y = f(x)$ is given to the right. Which of the following statements are **TRUE** and which are **FALSE**?



a. $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^-} f(x)$ _____

b. $\lim_{x \rightarrow 1} f(x)$ does not exist. _____

c. $\lim_{x \rightarrow 1^-} f(x) = 1$. _____

d. $\lim_{x \rightarrow 2} f(x)$ does not exist. _____

e. f is continuous and differentiable at $x = 0$. _____

f. f is not continuous at $x=1$, because $f(1)$ does not exist. _____

g. $\lim_{x \rightarrow 2} f(x) = f(2)$ _____

h. On the interval $-1 < x < 0$, $f' < 0$ and $f'' < 0$. _____

2. (4 points each) Evaluate the following limits. You may use any method, but show work where appropriate.

a. $\lim_{x \rightarrow 0} \frac{\cos 4x - 1}{\sin 4x} =$

b. $\lim_{x \rightarrow \infty} \frac{1 - 4x^2}{3x^2 - 2x + 1} =$

c. $\lim_{x \rightarrow 0^+} (x \cdot \ln x) =$

2. (continued)

d. $\lim_{x \rightarrow \infty} (1+x^2)^{\frac{1}{x}}$

3. (4 points each) Evaluate the derivatives of the following functions. You do not have to simplify your answers.

a. $y = 3x^2 - e^{-x} + \cos 3x$

b. $y = x^{\frac{2}{3}} \cdot \sin x$

c. $y = (\ln(3x+1))^3$

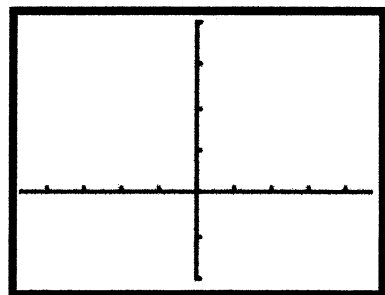
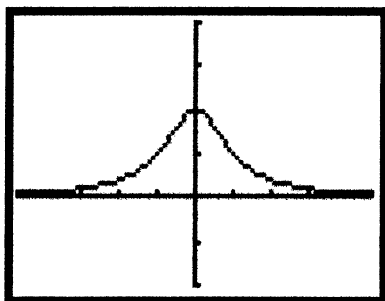
d. $y = \frac{x^4}{1 - \sin x}$

4. (5 points each) Evaluate the derivative y' of the following. In each case, solve for y' .

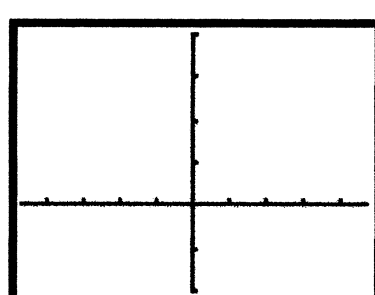
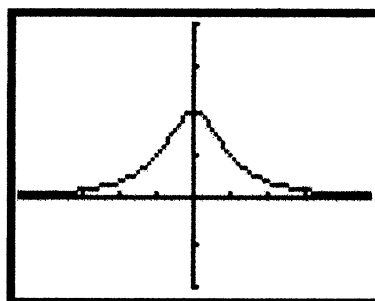
a. $x^3 - y^3 = xy^2 + 1$

b. $y = x^{\tan x}$, $x > 0$

5. (4 points) Given the graph of f shown below, sketch the graph of f' .

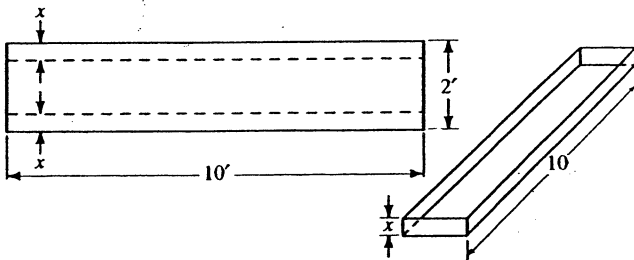


6. (4 points) Given the graph of f' (the derivative) shown below, sketch the graph of f .



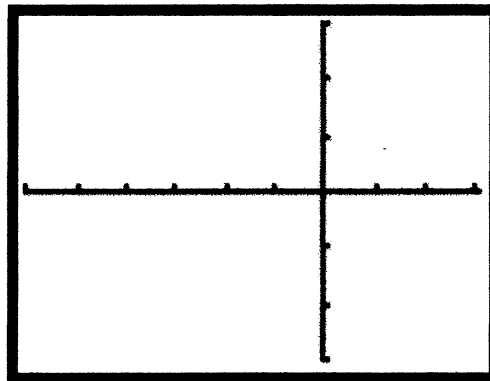
7. (6 points) A plane flying horizontally at an altitude of 1 mile with a speed of 500 miles per hour passes directly over a radar station. After the plane has passed directly over the radar station, find the rate at which the distance between the plane and the radar station is increasing when the distance between the plane and the station is 2 miles.

8. (6 points) A water trough is to be formed from a piece of metal 2 feet wide and 10 feet long by bending up at a right angle equal amounts from the long side. (Pieces will then be welded onto the ends.) How many inches on each side should be bent up to obtain the maximum volume of the water trough?



9. (6 points) Draw a graph of a function $y = f(x)$ that has all of the following characteristics.

- i) f is continuous on $(-\infty, \infty)$.
- ii) $f(-4) = 0$, $f(0) = 0$
- iii) $f'(x) > 0$ on $(-\infty, -4)$ and $(-2, \infty)$
 $f'(x) < 0$ on $(-4, -2)$
- iv) $f''(x) > 0$ on $(-\infty, -4)$ and $(-4, 0)$
 $f''(x) < 0$ on $(0, \infty)$



10. Given $f(x) = \frac{x^2}{x^2 - 1}$, $f'(x) = \frac{-2x}{(x^2 - 1)^2}$, and $f''(x) = \frac{2(3x^2 + 1)}{(x^2 - 1)^3}$

a. (3 points) Write equations of all asymptotes of f . _____

b. (4 points) Find intervals where f is increasing and decreasing.

Increasing: _____

Decreasing: _____

c. (4 points) Find intervals where the function is concave up and concave down.

Concave Up: _____

Concave Down: _____

d. (4 points) State x -values, if any, of all local maximum, local minimum, and inflection points. If there are none, write NONE.

Local maximum: _____ Local minimum: _____ Inflection points: _____

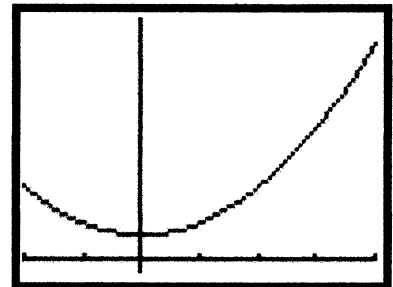
11. (4 points each) Given the parametric equations: $x = t^2 - t + 2$ and $y = \frac{1}{3}t^3 - t^2 + 1$.

a. Find the slope of the tangent line at $t = 3$.

b. Find all points (ordered pairs) on the curve where the tangent line is horizontal.

12. (4 points each) To the right is the graph of $f(x) = x^2 + 2$ on $[-2, 4]$.

a. Write the equation of the tangent line to f at $x=2$.



b. Estimate the area under the graph of f from $x = -2$ to $x = 4$ by using three approximating rectangles and right endpoints. Sketch the rectangles on the graph.

c. Set up the definite integral that would be used to calculate the area under f from $x = -2$ to $x = 4$ exactly, and evaluate the integral using the Fundamental Theorem of Calculus.

13. (5 points) If $f'(x) = e^x + 6x - 2\sin x$ and $f(0) = 5$, find the function $f(x)$.

14. (4 points each) Evaluate the following indefinite integrals.

a. $\int \frac{x^2 + x + 1}{x^2} dx$

b. $\int \frac{x^2}{x^3 - 1} dx$

15. (5 points each) Evaluate the following definite integrals.

a. $\int_0^{\pi/3} (1 + 2 \sin x) dx$

b. $\int_1^4 (x^2 + \sqrt{x}) dx$

c. $\int_0^{\pi/4} \sin^2 x \cos x dx$

16. (3 points) If $h(x) = \int_5^x t^2 \ln t dt$, then $h'(x) =$ _____