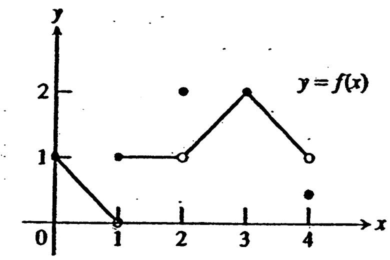
Name ____________________________

1. **(2 points each)** The complete graph of y = f(x) is given to the right. Which of the following statements are **TRUE** and which are **FALSE**?



a. $\lim_{x\to 0^+} f(x) = 1$ ____________

b. $\lim_{x\to 1^-} f(x) = 1$ ____________

c. $\lim_{x\to 1} f(x)$ does not exist. ____________

d. $\lim_{x\to 2} f(x)$ does not exist. ____________

e. $\lim_{x\to 4^-} f(x) \neq f(4)$ ____________

f. f is not continuous at x = 2 because f(2) does not exist. ____________

g. f is continuous and differentiable at x = 3. ____________

2. **(5 points)** Use the <u>definition of the derivative</u>, to find the derivative f'(x), if $f(x) = \dfrac{2}{x}$

3. **(4 points each)** Evaluate the following limits. Show all work.

a. $\lim_{x\to 0} \frac{e^x - e^{-x}}{\sin x}$

c. $\lim_{x\to 0^+} (x^2 \ln x)$

d. $\lim_{x\to 0^+} (1+2x)^{\frac{1}{3x}} =$

4. **(4 points)** Since $f(x) = x^3 + 5x^2 - 2x$ satisfies the hypothesis of the Mean Value Theorem on the interval [-1 , 2], find all values of x in the interval that satisfy the conclusion of the Mean Value Theorem.

5. **(4 points each)** Evaluate the derivatives of the following functions. **You do not have to simplify your answers.**

   a. $y = \dfrac{3}{x^3} - \sin^3 x$

   b. $y = x^2 \ln(x^2)$

   c. $y = \sqrt{\tan(5x-2)}$

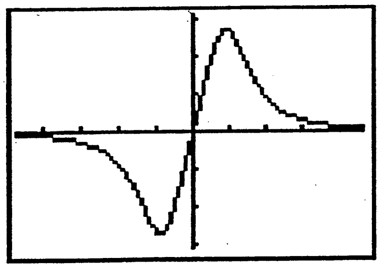
   d. $y = \dfrac{e^x - 1}{e^x + 1}$

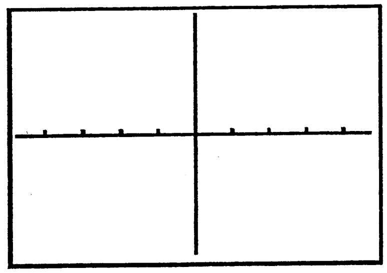
6. **(5 points each)** Evaluate the derivative **y'** of the following. In each case, solve for **y'**.

   a. $3x^2 - x^2y^2 = y^3 - 4$

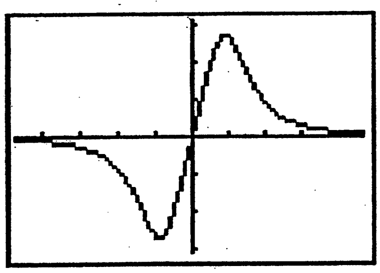
   b. $y = (x+1)^x$, $x > -1$

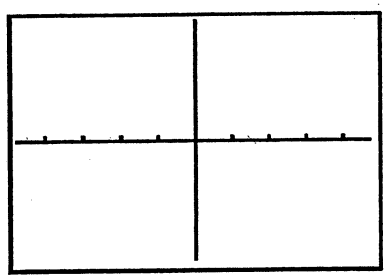
7. **(4 points)** Given the graph of **f** shown below, sketch the graph of **f'**.





8. **(4 points)** Given the graph of **f'** (the derivative) shown below, sketch the graph of **f**.



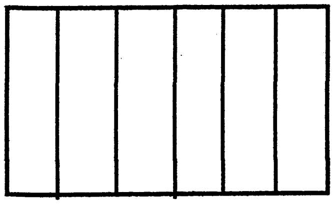


9. **(5 points each)** A ladder 13 ft long leans against a vertical building. The bottom of the ladder slides away from the building at a rate of 2 ft/sec.

a. How fast is the ladder sliding down the building when the top of the ladder is 5 ft above ground?

b. How fast is the angle between the ladder and the ground changing when the top of the ladder is 5 ft above ground?

10. **(6 points)** A veterinarian has 100 ft of fencing to construct six dog kennels by first building a fence around a rectangular region, and then subdividing that region into six smaller rectangles by placing five fences parallel to one of the sides. (See the figure below.) What dimensions of the total region will maximize the total area?



11. **(5 points)** Draw a graph of a function y = f(x) that has all of the following characteristics.

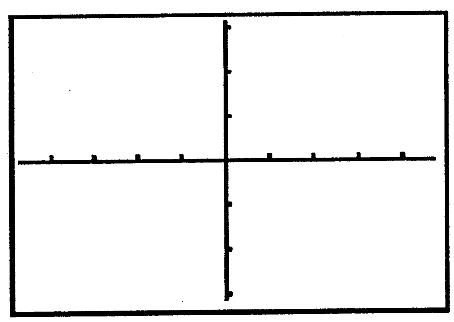
i) f is continuous on $(-\infty,\infty)$.

ii) $f(-2) = 0$, $f(2) = 0$, $f(0) = 2$

ii) $f'(0) = 0$, $f''(-1) = 0$, $f'(2)$ does not exist.

iii) $f'(x) > 0$ on $(-\infty,0)$ and $(2,\infty)$
$f'(x) < 0$ on $(0,2)$

iv) $f''(x) > 0$ on $(-\infty,-1)$
$f''(x) < 0$ on $(-1,2)$ and $(2,\infty)$



12. **(4 points)** If $f'(x) = 9x^2 + \frac{1}{x} + 3\sqrt{x}$ and f(1) = 5, find the function f(x).

13. Given $f(x) = 3x^4 - 8x^3$

a. **(4 points)** Find intervals where f is increasing and decreasing.

Increasing: ____________________

Decreasing: ____________________

b. **(4 points)** Find intervals where the function is concave up and concave down.

Concave Up: ____________________

Concave Down: __________________

c. **(6 points)** State x-values, if any, of all local maximum, local minimum, and inflection points. If there are none, write NONE.

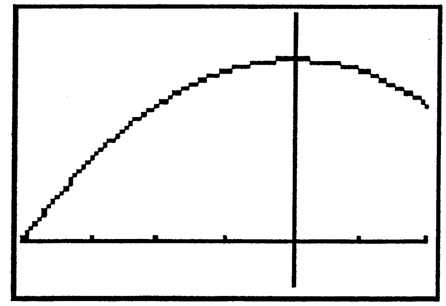
Local maximum: _________ Local minimum: ___________ Inflection points: ___________

14. **(4 points each)** Given the parametric equations: $x = t + 2\sin t$ and $y = t - 2\cos t$ on $[0, 2\pi]$.

a. Find the slope of the tangent line at $t = \pi/2$.

b. Find all <u>t-values</u> in $[0, 2\pi]$ where points on the curve have a vertical tangent line.

15. **(4 points each)** To the right is the graph of $f(x) = -\frac{1}{2}x^2 + 8$.



a. Estimate the area under the graph of f from x = -4 to x = 2 by using three approximating rectangles and right endpoints. Sketch the rectangles on the graph.

b. Set up the definite integral that would be used to calculate the area under f from x = -4 to x = 2 exactly, and evaluate the integral using the Fundamental Theorem of Calculus.

16. **(4 points each)** Evaluate the following indefinite integrals.

a. $\int \frac{x+3}{\sqrt{x}} dx$

b. $\int \frac{x}{x^2+4} dx$

17. **(5 points each)** Evaluate the following definite integrals.

a. $\int_{1}^{e}\left(2x+\frac{1}{x}\right)dx$

b. $\int_{\pi/3}^{\pi}(6+2\sin 2x)dx$

c. $\int_{1}^{2}xe^{x^2}dx$

18. **(3 points)** If $h(x)=\int_{-2}^{x}\sqrt{t}\tan t\,dt$, then h'(x)= ______________________________