MATH 1401 UNIFORM FINAL EXAM

December 6, 2003

Name: _____

Student Number: _____

Circle your section and instructor:

001	002	003	004	005	OL1
MacMillan Mon/Wed 9:00-10:50	MacMillan Mon/Wed 3:00-4:50	Ovtchinnikov Tue/Thur 11:00-12:50	Pendergrass Tue/Thur 1:00-2:50	Douthitt Tue/Thur 5:00-6:50	Byrne On Line

Directions:

1. Complete the section above.
2. Put your name on ***Page 1*** of the test. You should have **8** pages of this test.
3. Show all work and be neat! If we can not follow your work, you will not receive any credit.
4. If you are confused about what a problem is asking, ask your instructor. You may not ask for hints or a verification on how you have completed a problem.
5. You are allowed 1/2 sheet of notes. You are not allowed calculators or computers.

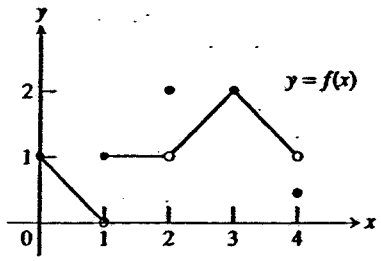
DO NOT WRITE IN THIS SPACE

1. Page 1 (22 pts)	2. Page 2 (16 pts)	3. Page 3 (26 pts)	4. Page 4 (14 pts)
5. Page 5 (16 pts)	6. Page 6 (22 pts)	7. Page 7 (16 pts)	8. Page 8 (18 pts)

TOTAL: _____ (out of 150 points)

Name _____

1. **(2 points each)** The complete graph of y = f(x) is given to the right. Which of the following statements are **TRUE** and which are **FALSE**?



a. $\lim_{x\to 0^+} f(x) = 1$ _____

b. $\lim_{x\to 1^-} f(x) = 1$ _____

c. $\lim_{x\to 1} f(x)$ does not exist. _____

d. $\lim_{x\to 2} f(x)$ does not exist. _____

e. $\lim_{x\to 4^-} f(x) \neq f(4)$ _____

f. f is not continuous at x = 2 because f(2) does not exist. _____

g. f is continuous and differentiable at x = 3. _____

h. The domain of f is [0 , 4]. _____

2. **(3 points each)** Solve the equations for x. Show your work.

a. $7 - 3e^{2x} = 1$

b. $1 + 2\ln(3x) = 5$

3. **(4 points each)** Evaluate the following limits. You may use any method, but show work where appropriate.

a. $\lim_{x\to 0} \dfrac{e^x - e^{-x}}{\sin x}$

b. $\lim_{x\to\infty} \dfrac{3-7x^2}{4x^2-5x+10} =$

c. $\lim_{x\to 0^+} (x^2 \ln x)$

d. $\lim_{x\to 0^+} (1+2x)^{\frac{1}{3x}} =$

4. **(4 points each)** Evaluate the derivatives of the following functions. **You do not have to simplify your answers.**

a. $y = \frac{3}{x^3} - \sin^3 x$

b. $y = x^2 \ln(x^2)$

c. $y = \sqrt{\tan(5x-2)}$

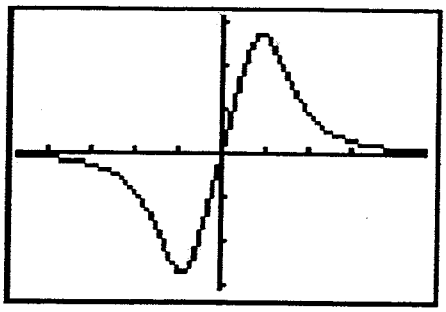
d. $y = \frac{e^x - 1}{e^x + 1}$

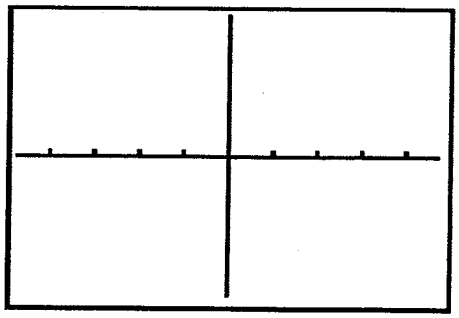
5. **(5 points each)** Evaluate the derivative **y'** of the following. In each case, solve for **y'**.

a. $3x^2 - x^2y^2 = y^3 - 4$

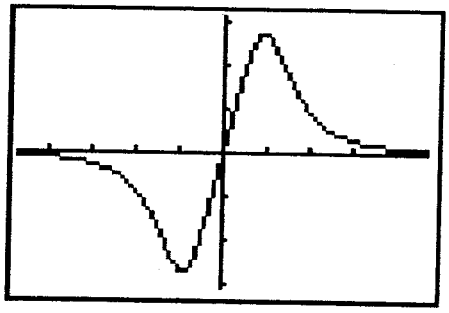
b. $y = (x+1)^x, x > -1$

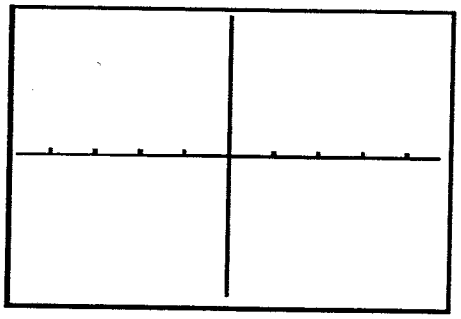
6. **(4 points)** Given the graph of **f** shown below, sketch the graph of **f'**.





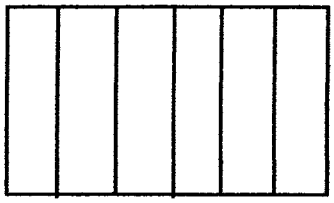
7. **(4 points)** Given the graph of **f'** (the derivative) shown below, sketch the graph of **f**.





8. **(6 points)** A ladder 20 ft long leans against a vertical building. If the bottom of the ladder slides away from the building at a rate of 2 ft/sec, how fast is the ladder sliding down the building when the top of the ladder is 16 ft above ground?

9. **(6 points)** A veterinarian has 100 ft of fencing to construct six dog kennels by first building a fence around a rectangular region, and then subdividing that region into six smaller rectangles by placing five fences parallel to one of the sides. (See the figure below.) What dimensions of the region will maximize the total area?



10. **(5 points)** Draw a graph of a function $y = f(x)$ that has all of the following characteristics.

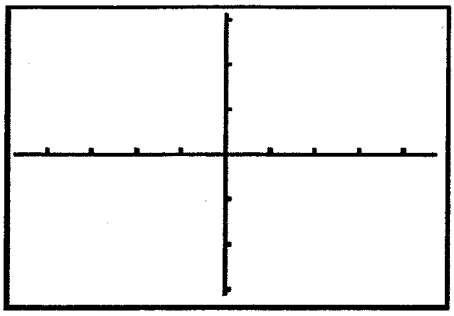
i) f is continuous on $(-\infty,\infty)$.

ii) $f(-2) = 0,\ f(2) = 0,\ f(0) = 2$

ii) $f'(0) = 0,\ f''(-1) = 0,\ f'(2)$ does not exist.

iii) $f'(x) > 0$ on $(-\infty,0)$ and $(2,\infty)$
$f'(x) < 0$ on $(0,2)$

iv) $f''(x) > 0$ on $(-\infty,-1)$
$f''(x) < 0$ on $(-1,2)$ and $(2,\infty)$



11. **(5 points)** If $f'(x) = 9x^2 + \frac{1}{x} + 3\sqrt{x}$ and $f(1) = 5$, find the function $f(x)$.

12. Given $f(x) = 3x^4 - 8x^3$

a. **(4 points)** Find intervals where f is increasing and decreasing.

Increasing: _____

Decreasing: _____

b. **(4 points)** Find intervals where the function is concave up and concave down.

Concave Up: _____

Concave Down: _____

c. **(6 points)** State x-values, if any, of all local maximum, local minimum, and inflection points. If there are none, write NONE.

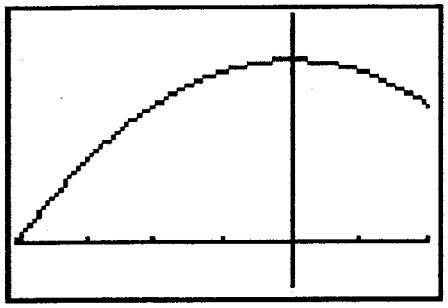
Local maximum: _____ Local minimum: _____ Inflection points: _____

13. **(4 points each)** Given the parametric equations: $x = t + 2\sin t$ and $y = t - 2\cos t$ on $[0, 2\pi]$.

a. Find the slope of the tangent line at $t = \pi/2$.

b. Find all t-values in $[0, 2\pi]$ where points on the cueve have a vertical tangent line.

14. **(4 points each)** To the right is the graph of $f(x) = -\frac{1}{2}x^2 + 8$.



a. Estimate the area under the graph of f from x = -4 to x = 2 by using three approximating rectangles and right endpoints. Sketch the rectangles on the graph.

b. Set up the definite integral that would be used to calculate the area under f from x = -4 to x = 2 exactly, and evaluate the integral using the Fundamental Theorem of Calculus.

15. **(4 points each)** Evaluate the following indefinite integrals.

a. $\int \frac{x+3}{\sqrt{x}}\,dx$

b. $\int \frac{x}{x^2+4}\,dx$

16. **(5 points each)** Evaluate the following definite integrals.

a. $\int_1^e \left(2x+\frac{1}{x}\right)dx$

b. $\int_{\pi/2}^{\pi}(6-2\cos x)dx$

c. $\int_1^2 xe^{x^2}dx$

17. **(3 points)** If $h(x)=\int_{-2}^{x}\sqrt{t}\tan t\,dt$, then $h'(x)=$ _____