# MATH 1401 UNIFORM FINAL EXAM

**December 7,2002**

Name: ______________________________

Student Number: ____________________

Circle your section and instructor:

| 001 | 002 | 003 | 004 | 005 | OL1 |
|---|---|---|---|---|---|
| B. MacMillan<br>Mon/Wed<br>9:00-10:50 | B. MacMillan<br>Mon/Wed<br>3:00-4:50 | J. Matsuo<br>Tue/Thur<br>11:00-12:50 | D. Stewart<br>Tue/Thur<br>1:00-2:50 | J. Shapiro<br>Tue/Thur<br>5:00-6:50 | R. Byrne<br>On Line |

Directions:

1. Complete the section above.
2. Put your name on ***Page 1*** of the test. You should have **8** pages of this test.
3. Show all work and be neat! If we can not follow your work, you will not receive any credit.
4. If you are confused about what a problem is asking, ask your instructor. You may not ask for hints or a verification on how you have completed a problem.
5. You are allowed 1/2 sheet of notes. You are not allowed calculators or computers.

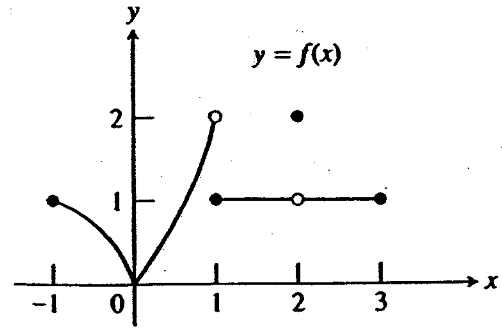
**DO NOT WRITE IN THIS SPACE**

| | | | |
|---|---|---|---|
| 1. Page 1 (21 pts) | 2. Page 2 (16 pts) | 3. Page 3 (26 pts) | 4. Page 4 (14 pts) |
| 5. Page 5 (16 pts) | 6. Page 6 (23 pts) | 7. Page 7 (16 pts) | 8. Page 8 (18 pts) |

TOTAL: ____________________ (out of 150 points)

**Name** ________________________________________

1. **( 2 points each )** The complete graph of $y = f(x)$ is given to the right. Which of the following statements are **TRUE** and which are **FALSE**?




a. $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x)$ ________________

b. $\lim_{x\to 1} f(x)$ does not exist. ________________

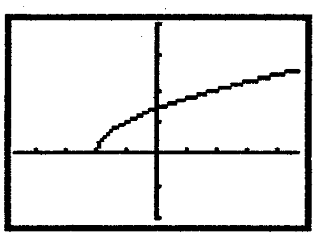
c. $\lim_{x\to 1^-} f(x)$ does not exist. ________________

d. $\lim_{x\to 2} f(x)$ does not exist. ________________

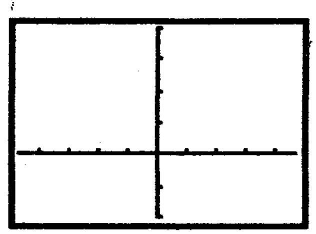
e. f is continuous and differentiable at $x = 0$. ________________

2. **( 3 points each )** The graph of a function f is given below. Use it to graph the indicated functions.

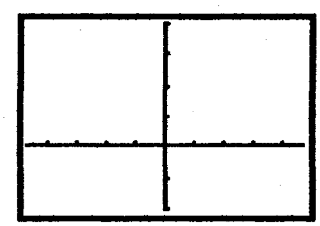
$y = f(x)$



a. $y = -f(x-2)+1$



$y = f^{-1}(x)$ (The inverse of $f(x)$)



3. **( 5 points )** Solve the equation for x, such that $x>0$. Show your work.

$$\log_2 x + \log_2 (x-3) = 2$$

4. **( 4 points each )** Evaluate the following limits. You may use any method, but show work where appropriate.

a. $\lim_{x\to 3}\dfrac{x-3}{3x^2-7x-6}=$

b. $\lim_{x\to\infty}\dfrac{1-4x^2}{3x^2-2x+1}=$

c. $\lim_{x\to 0^+}(x\cdot\ln x)=$

d. $\lim_{x\to 0}\left(e^x+x\right)^{\frac{1}{x}}=$

5. **( 4 points each )** Evaluate the derivatives of the following functions. **You do not have to simplify your answers.**

a. $y = 3x^2 - e^{-x} + \cos 3x$

b. $y = x^2 \tan^{-1} x$

c. $y = (\ln(3x+1))^3$

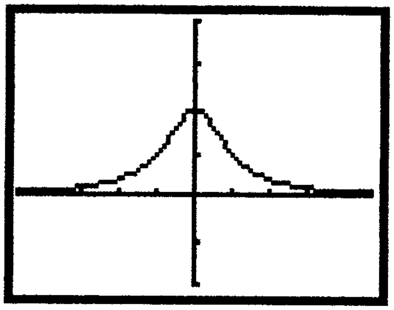
d. $y = \dfrac{x^4}{1 - \sin x}$

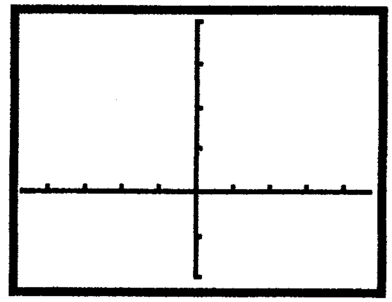
6. **( 5 points each )** Evaluate the derivative **y'** of the following. In each case, solve for **y'**.

a. $x^3 - y^3 = xy^2 + 1$

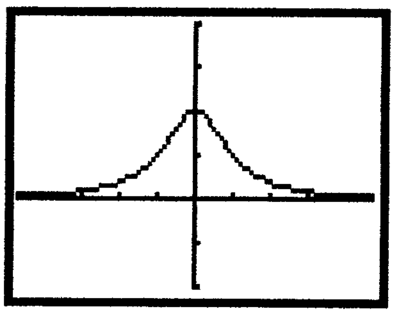
b. $y = x^{\tan x}$ , $x>0$

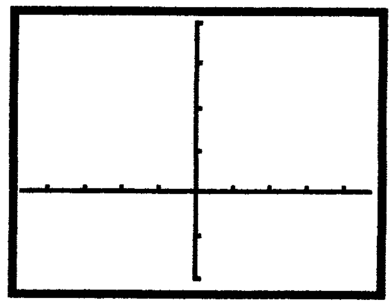
7. **( 4 points )** Given the graph of **f** shown below, sketch the graph of **f'**.





8. **( 4 points )** Given the graph of **f'** (the derivative) shown below, sketch the graph of **f**.





9. **( 6 points )** A plane flying horizontally at an altitutude of 1 mile and a speed of 500 miles per hour passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when the plane is 2 miles away from the station?

10. **( 6 points )** A rectangular box with an open top and a square base is to be made out of 1200 $cm^2$ of material. Find the dimensions of the box that will maximize the volume of the box.

11. **( 5 points )** Draw a graph of a function $y = f(x)$ that has all of the following characteristics.

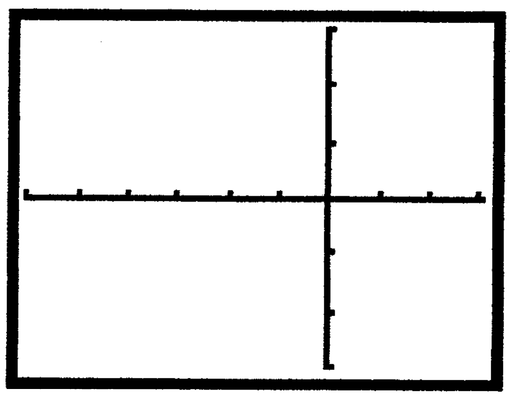
i) $f$ is continuous on $(-\infty, \infty)$.

ii) $f(-4) = 0, \; f(0) = 0$

ii) $f'(-2) = 0, \; f''(0) = 0, \; f'(-4)$ does not exist.

iii) $f'(x) > 0$ on $(-\infty, -4)$ and $(-2, \infty)$
$f'(x) < 0$ on $(-4, -2)$

iv) $f''(x) > 0$ on $(-\infty, -4)$ and $(-4, 0)$
$f''(x) < 0$ on $(0, \infty)$



12. **( 5 points )** If $f'(x) = e^x + 6x - 2\sin x$ and $f(0) = 5$, find the function $f(x)$.

13. Given $f(x)=\dfrac{x^2}{x^2-1}$ , $f'(x)=\dfrac{-2x}{\left(x^2-1\right)^2}$ , and $f''(x)=\dfrac{2(3x^2+1)}{\left(x^2-1\right)^3}$

a. **( 3 points )** Write equations of all asymptotes. ______________________________

b. **( 4 points )** Find intervals where f is increasing and decreasing.

Increasing: ____________________

Decreasing: ____________________

c. **( 4 points )** Find intervals where the function is concave up and concave down.

Concave Up: ____________________

Concave Down: __________________

d. **( 4 points )** State x-values, if any, of all local maximum, local minimum, and inflection points. If there are none, write NONE.

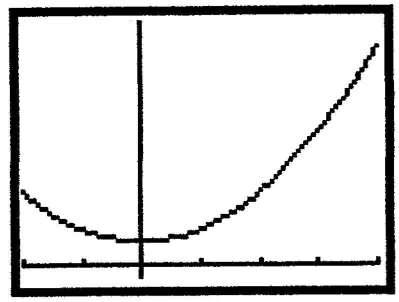
Local maximum: _________ Local minimum: ___________ Inflection points: ___________

14. **( 4 points each )** Given the parametric equations: $x=t^2-t+2$ and $y=\frac{1}{3}t^3-t^2+1$.

a. Find the slope of the tangent line at $t=3$.

b. Find all points on the curve where the tangent line is horizontal.

15. **( 4 points each )** To the right is the graph of $f(x) = x^2 + 2$ on [ -2 , 4 ].



a. Estimate the area under the graph of f from x = -2 to x = 4 by using three approximating rectangles and right endpoints. Sketch the rectangles on the graph.

b. Set up the definite integral that would be used to calculate the area under f from x = -2 to x = 4 exactly, and evaluate the integral using the Fundamental Theorem of Calculus.

16. **( 4 points each )** Evaluate the following indefinite integrals.

a. $\int \frac{x^2 + x + 1}{x^2} dx$

b. $\int \frac{x^2}{x^3 - 1} dx$

17. **( 5 points each )** Evaluate the following definite integrals.

a. $\int_0^{\pi/3} (1+2\sin x)dx$

b. $\int_1^4 \left(x^2+\sqrt{x}\right)dx$

c. $\int_0^{\pi/4} \sin^2 x \cos x \, dx$

18. **( 3 points )** If $h(x)=\int_5^x t^2 \ln t \, dt$, then $h'(x)=$ ______________________