

# MATH 1120 UNIFORM FINAL EXAM

May 10<sup>th</sup>, 2008

Name: \_\_\_\_\_

Student Number: \_\_\_\_\_

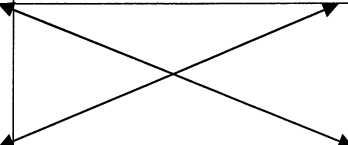
Circle Your Section and Instructor:

001	002	003
Nadia Hamoudi M/W 4:00-5:15	Erik Heiny M/W 1:00-2:15	Burt Simon T/TH 8:30-9:45

## Directions:

1. Complete the Section Above.
2. Put your name on page 1 of the test. You should have 6 pages of test questions.
3. This exam is closed calculator, closed note, and closed book.
4. If you are confused about what a problem is asking, ask your instructor. You may not ask for hints or a verification on how you have completed a problem.

Do Not Write In This Space

Page 1 (29 Points)	Page 2 (20 Points)	Page 3 (19 Points)	Page 4 (22 Points)
Page 5 (14 Points)	Page 6 (24 Points)	Page 7 (22 Points)	

Total: \_\_\_\_\_ (Out of 150 Points)

Name : \_\_\_\_\_ Date: \_\_\_\_\_

1. (18 pts.) Find the exact value of each expression. Circle your final answer.

a)  $\cos\left(\frac{4\pi}{3}\right)$

d)  $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

b)  $\tan\left(\frac{8\pi}{3}\right)$

e)  $\sec\left(\frac{5\pi}{4}\right)$

c)  $\sin(-240^\circ)$

f)  $\sin\left(\frac{5\pi}{6}\right)$

2. (3 pts) Convert the angle whose measure is  $80^\circ$  to radians. Simplify your answer.

3. (6 pts.) The terminal side of angle  $\theta$  passes through (6, -8). Find the exact value of each trigonometric function of  $\theta$ . Simplify your answers.

$\sin(\theta) =$

$\tan(\theta) =$

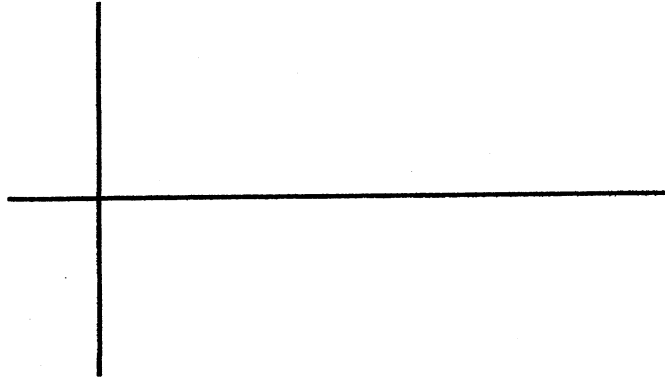
$\sec(\theta) =$

4. (2 pts.) If  $\cos(\theta) < 0$  and  $\tan(\theta) < 0$ , then  $\theta$  lies in Quadrant \_\_\_\_\_.

5. (6 pts.) Sketch the graph of  $y = \cot(x)$  on the interval  $[0, 2\pi]$ . Show asymptotes with dotted lines. State the domain and range.

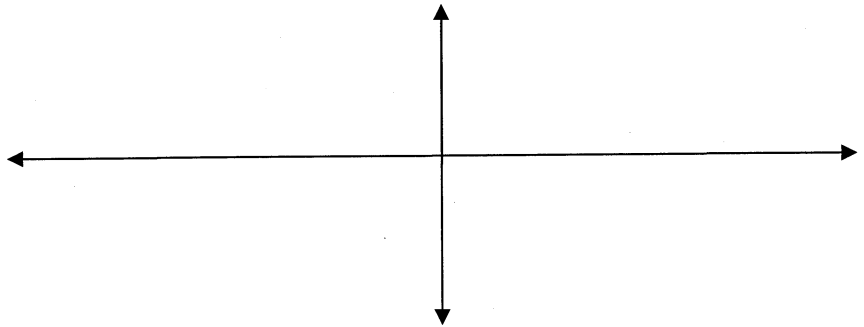
Domain: \_\_\_\_\_

Range: \_\_\_\_\_



6. (7 pts. Each) Sketch the graph (at least one period) of each function below and state the amplitude, period, vertical shift, and phase shift. Label the important values on the axes.

a.  $y = -2\cos(x) + 1$



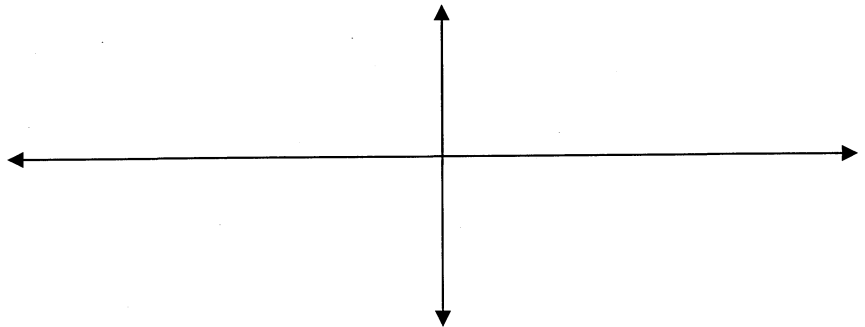
Amplitude: \_\_\_\_\_

Period: \_\_\_\_\_

Vertical Shift: \_\_\_\_\_

Phase Shift: \_\_\_\_\_

b)  $y = 4\sin(3x - \pi)$



Amplitude: \_\_\_\_\_

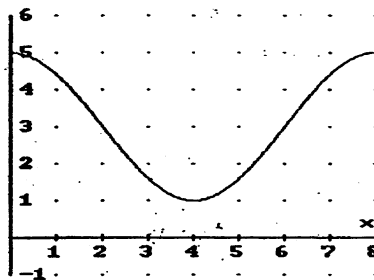
Period: \_\_\_\_\_

Vertical Shift: \_\_\_\_\_

Phase Shift: \_\_\_\_\_

7. (5 pts.) One period of a trig function is shown below. Write an equation for the function.

y = \_\_\_\_\_



8. (5 pts. each). Prove the following identities. Show all steps.

a.  $\tan \theta + \cot \theta = \sec \theta \csc \theta$

b.  $\frac{\cos \theta}{1 + \sin \theta} = \sec \theta - \tan \theta$

9. (4 pts.) Find the exact value of  $\sin\left(\frac{11\pi}{12}\right)$ .

10. (4 pts. each). If  $\sin \alpha = \frac{3}{5}$ ,  $\frac{\pi}{2} < \alpha < \pi$ ;  $\cos \beta = \frac{-5}{13}$ ,  $\frac{\pi}{2} < \beta < \pi$ , find the exact value of:

a.  $\sin(\alpha + \beta) =$

b.  $\cos(2\beta) =$

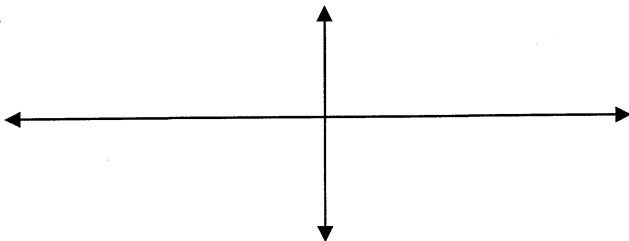
11. (4 pts.) Solve the following trigonometric equation on the interval  $[0, 2\pi]$ . Show your work.

$$\sin 2\theta + \cos \theta = 0$$

$$\theta = \underline{\hspace{4cm}}$$

12. (2 pt.) Sketch the graph of  $f(x) = \cos x$  on  $[-2\pi, 2\pi]$ .

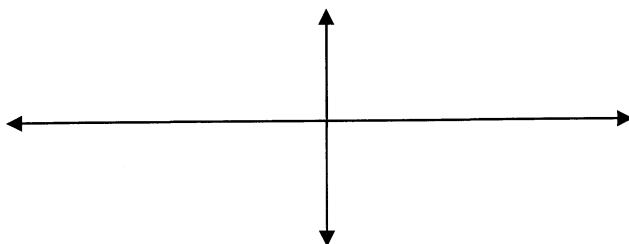
a.



b. (2 pts.) The function  $f$  is not a one-to-one function. To make it a one-to-one function, restrict the domain to:

$$[ \underline{\hspace{2cm}}, \underline{\hspace{2cm}} ]$$

c. (4 pts.) The inverse of  $f(x)$  is  $f^{-1}(x) = \cos^{-1}(x)$ . Sketch the graph of  $f^{-1}$  and state the **domain** and **range**.



Domain:  $\underline{\hspace{4cm}}$

Range:  $\underline{\hspace{4cm}}$

d. (2 pts.) Find:  $\cos^{-1}(-1) = \underline{\hspace{4cm}}$

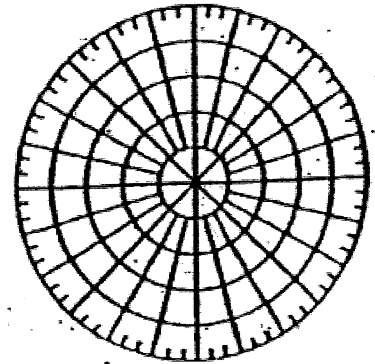
13. Given the point  $(-4, \frac{5\pi}{4})$  in polar coordinates.

a. (2 pts.) Plot the point.

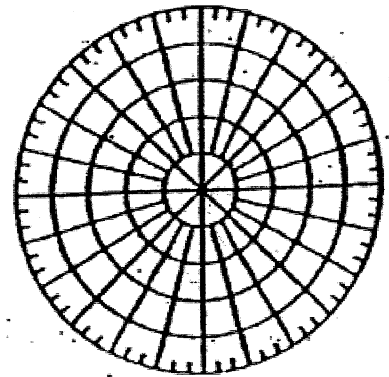
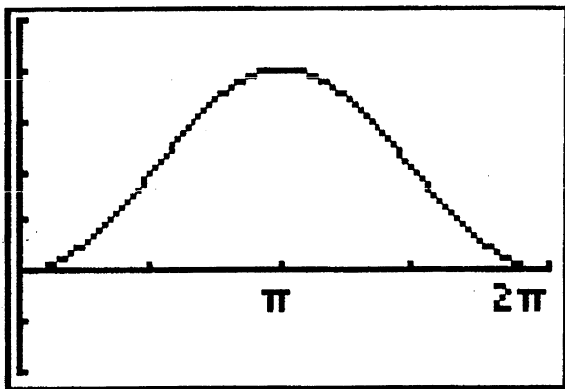
b. (2 pts.) Find other polar coordinates  $(r, \theta)$  of the point for which  $r > 0$  and  $0 < \theta < 2\pi$ .

( \_\_\_\_\_, \_\_\_\_\_ )

c. (2 pts.) Find the rectangular coordinates of the point. ( \_\_\_\_\_, \_\_\_\_\_ )



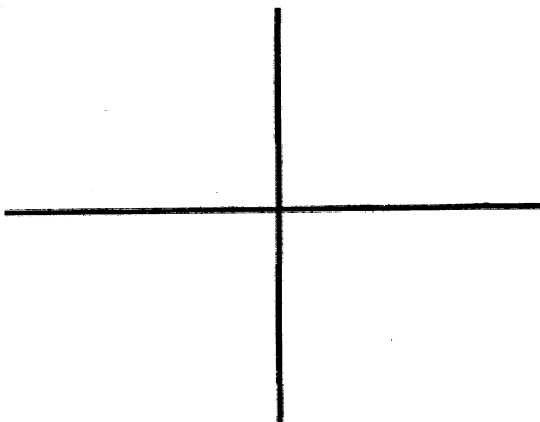
14. (4 pts.) Given the polar equation  $r = 2 - 2 \cos \theta$  and its corresponding graph in rectangular form. Sketch the graph of the equation in polar equations.



15. Given the complex number  $-6 + 6i$ ,

a. (1 pt.) Plot the complex number in the complex number plane.

b. (3 pts.) Write the complex number in polar form.



16. Given the complex number in polar form  $z = 2(\cos 120^\circ + i \sin 120^\circ)$

a. (4pts.) Write  $z$  in rectangular form.

b. (4 pts.) Calculate and simplify:  $z^5 =$   
(Note: Simplify your answer, but you may leave it in polar form.)

17. (4 pts.) If you graphed the parametric equations  $x = \sqrt{t} + 4$  and  $y = -4t^2$ , on the interval  $[1, 4]$  the starting point of the graph would be ( \_\_\_\_\_ , \_\_\_\_\_ ) and the ending point would be ( \_\_\_\_\_ , \_\_\_\_\_ ).

18. (4 pts. each). Write the first 4 terms of the sequences defined as:

a.  $\left\{ (-1)^{n+1} \cdot \frac{n}{n+1} \right\}$  \_\_\_\_\_ , \_\_\_\_\_ , \_\_\_\_\_ , \_\_\_\_\_

b.  $a_1 = -2$ ,  $a_n = n \cdot a_{n-1}$  \_\_\_\_\_ , \_\_\_\_\_ , \_\_\_\_\_ , \_\_\_\_\_

19. (4 pts.) Write a rule for the  $n$ th term of the sequence that begins,  $\frac{1}{3}, \frac{3}{6}, \frac{5}{9}, \frac{7}{12}, \dots$

$a_n =$  \_\_\_\_\_

20. (5 pts.) Write the first two terms and the twelfth term of the series. Then, using the properties of sigma notation, find the sum of the series.

$$\sum_{k=1}^{12} (k^2 + 3k - 6) = \underline{\hspace{2cm}} + \underline{\hspace{2cm}} + \dots \underline{\hspace{2cm}} =$$

21. (5 pts.) The angle of elevation to the top of a building from a point 300 ft. away from the base of the building on level ground is  $30^\circ$ . Find the height of the building.

22. (4 pts. each) Find the indicated part of the triangles below.

a. If  $\alpha = 30^\circ$ ,  $\beta = 105^\circ$ , and  $a=7$ , find  $c$ .

b. If  $a=12$ ,  $b=4$ , and  $\alpha = 120^\circ$ , find  $\beta$ . (Write your answer in terms of an inverse trig function.)

c. If  $\alpha = 60^\circ$ ,  $b=6$ , and  $c=8$ , find  $a$ .

**Sum and Difference Identities**

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

**Double Angle Identities**

$$\sin 2u = 2 \sin u \cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u = 2 \cos^2 u - 1 = 1 - 2 \sin^2 u$$

**Polar Coordinates**  $(x, y) \leftrightarrow (r, \theta)$ 

$$x = r \cos \theta$$

$$r = \sqrt{x^2 + y^2}$$

And

$$y = r \sin \theta$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

**Complex Numbers**  $x + yi \leftrightarrow r(\cos \theta + i \sin \theta)$ **DeMoivre's Theorem**

$$\text{If } z = r(\cos \theta + i \sin \theta), \text{ then } z^n = r^n(\cos(n\theta) + i \sin(n\theta))$$

**Law of Sines**

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

**Law of Cosines**

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

**Properties of Series**

$$1. \sum_{k=1}^n c = c \cdot n$$

$$2. \sum_{k=1}^n c \cdot a_k = c \cdot \sum_{k=1}^n a_k$$

$$3. \sum_{k=1}^n k = \frac{n(n+1)}{2}$$

$$4. \sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

**Graphing Trig Functions**

$$y = A \sin(\omega x - \phi) \text{ or } y = A \cos(\omega x - \phi)$$

$$\text{Amplitude} = |A| \quad \text{Period} = T = \frac{2\pi}{\omega} \quad \text{Phase Shift} = \frac{\phi}{\omega}$$