

MATH 1120 UNIFORM FINAL EXAM

May 6th, 2006

Name: _____

Student Number: _____

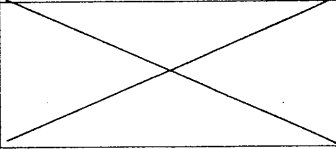
Circle Your Section and Instructor:

001	002
S. Boerckel Mon/Wed 5:30-6:45	A. Harris Tue/Thur 1:00-2:15

Directions:

1. Complete the Section Above.
2. Put your name on page 1 of the test. You should have 6 pages of test questions.
3. A graphing calculator will be allowed only on the last page of the exam. You will receive this page once you complete and turn in the non-calculator portion.
4. If you are confused about what a problem is asking, ask your instructor. You may not ask for hints or a verification on how you have completed a problem.

Do Not Write In This Space

Page 1 (31 points)	Page 2 (19 points)	Page 3 (17 points)	Page 4 (21 points)
Page 5 (14 points)	Page 6 (22 points)	Page 7 (26 points)	

Total: _____ (Out of 150 Points)

1. (20 pts.) Evaluate the following. If you receive less than 90% on this problem (18 pts.), your final exam grade will be lowered by one letter grade.

$$\sin\left(\frac{3\pi}{4}\right)$$

$$\cos\left(\frac{13\pi}{4}\right)$$

$$\tan\left(\frac{8\pi}{3}\right)$$

$$\sin\left(\frac{-2\pi}{3}\right)$$

$$\cos\left(\frac{4\pi}{3}\right)$$

$$\cos(210^\circ)$$

$$\sin(-240^\circ)$$

$$\tan(225^\circ)$$

$$\sin(510^\circ)$$

$$\cos(315^\circ)$$

2. (3 pts) Convert the angle whose measure is $\frac{-5\pi}{4}$ radians to degrees. Simplify your answer.

3. (6 pts.) The terminal side of angle θ passes through $(-1, -2)$. Find the exact value of each trigonometric function of θ . Simplify your answers.

$$\cos(\theta) =$$

$$\tan(\theta) =$$

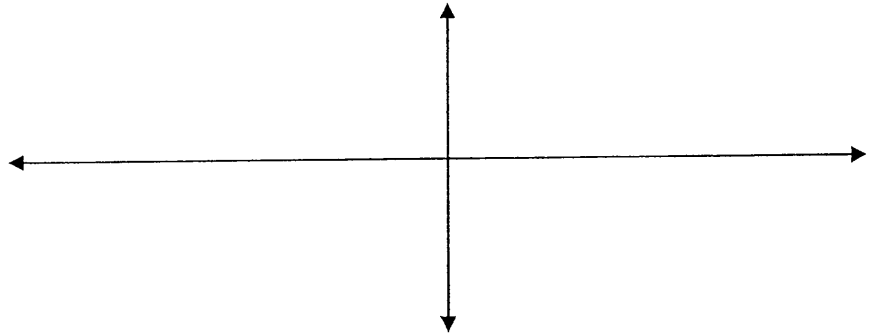
$$\cot(\theta) =$$

4. (2 pts.) If $\cos(\theta) < 0$ and $\tan(\theta) < 0$, then θ lies in Quadrant _____.

5. (5 pts.) Sketch the graph of $y = \tan x$ on the interval $[-\pi, \pi]$. Show asymptotes with dotted lines. State the domain and range.

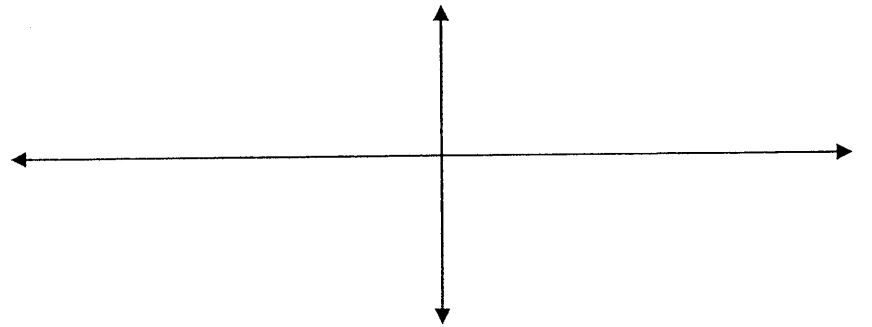
Domain: _____

Range: _____



6. (7 pts. Each) Sketch the graph (at least one period) of each function below and state the amplitude, period, vertical shift, and phase shift. Label the important values on the axes.

a. $y = 3 \cos(2x + \pi)$



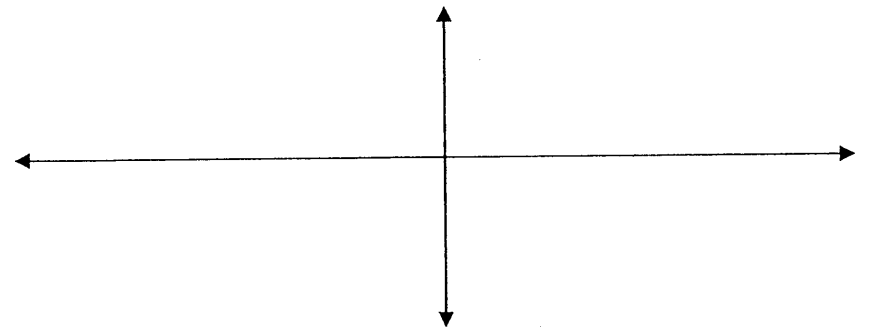
Amplitude: _____

Period: _____

Vertical Shift: _____

Phase Shift: _____

b) $y = -3 \sin(2x + \frac{\pi}{2})$



Amplitude: _____

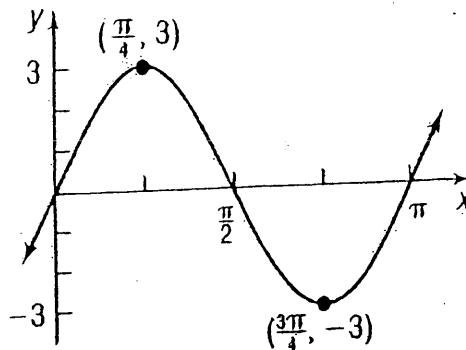
Period: _____

Vertical Shift: _____

Phase Shift: _____

7. (5 pts.) One period of a trig function is shown below. Write an equation for the function.

y = _____



8. (4 pts. each). Prove the following identities. Show all steps.

a. $\tan \theta + \cot \theta = \sec \theta \csc \theta$

b. $\frac{\cos \theta}{1 + \sin \theta} = \sec \theta - \tan \theta$

9. (4 pts.) Find the exact value of $\cos\left(\frac{7\pi}{12}\right)$.

10. (4 pts. each). If $\sin \alpha = \frac{3}{5}$, $0 < \alpha < \frac{\pi}{2}$; $\cos \beta = \frac{2\sqrt{5}}{5}$, $-\frac{\pi}{2} < \beta < 0$, find the exact value of:

a. $\cos(\alpha + \beta) =$

b. $\sin(2\beta) =$

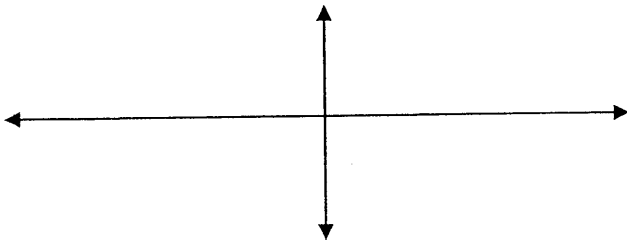
11. (4 pts.) Solve the following trigonometric equation on the interval $[0, 2\pi]$. Show your work.

$$4 \cos^2 \theta - 3 = 0$$

$$\theta = \underline{\hspace{4cm}}$$

12. (1 pt.) Sketch the graph of $f(x) = \cos x$ on $[-2\pi, 2\pi]$.

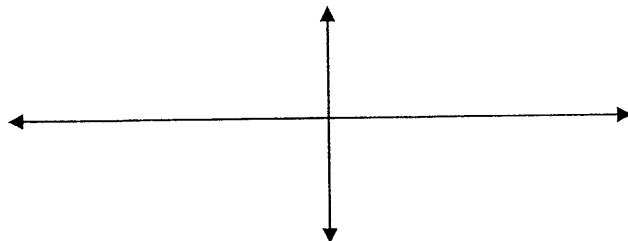
a.



b. (2 pts.) The function f is not a one-to-one function. To make it a one-to-one function, restrict the domain to:

$$[\underline{\hspace{2cm}}, \underline{\hspace{2cm}}]$$

c. (4 pts.) The inverse of $f(x)$ is $f^{-1}(x) = \cos^{-1}(x)$. Sketch the graph of f^{-1} and state the **domain** and **range**.



Domain: $\underline{\hspace{4cm}}$

Range: $\underline{\hspace{4cm}}$

d. (2 pts.) Find: $\cos^{-1}(-1) = \underline{\hspace{4cm}}$

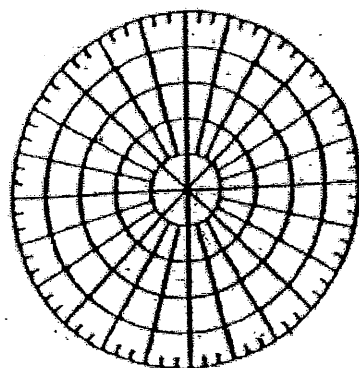
13. Given the point $(-2, \frac{-2\pi}{3})$ in polar coordinates.

a. (2 pts.) Plot the point.

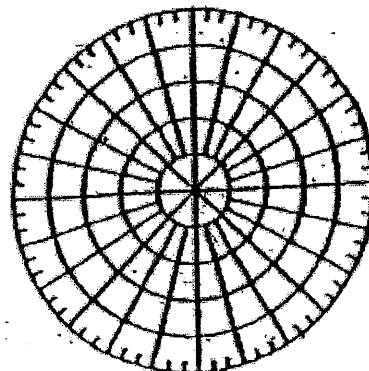
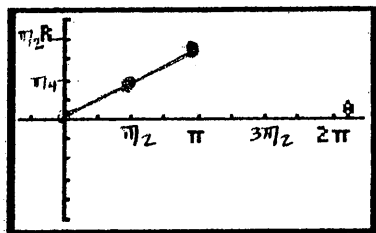
b. (2 pts.) Find other polar coordinates (r, θ) of the point for which $r > 0$ and $0 < \theta < 2\pi$.

(_____, _____)

c. (2 pts.) Find the rectangular coordinates of the point. (_____, _____)



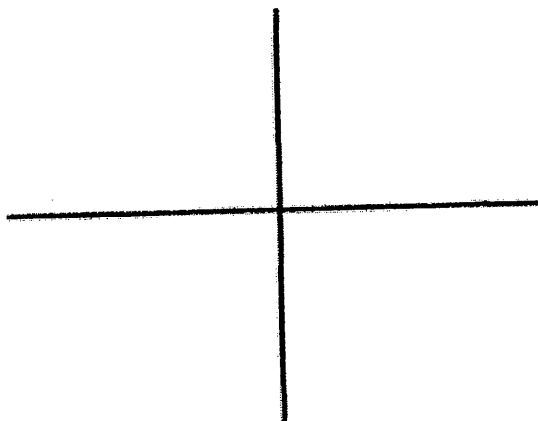
14. (4 pts.) Given the polar equation $r = \frac{\theta}{2}$ and its corresponding graph in rectangular form. Sketch the graph of the equation in polar coordinates from 0 to π . (Recall, $\pi \approx 3.14$)



15. Given the complex number $4 - 4i$,

a. (1 pt.) Plot the complex number in the complex number plane.

b. (3 pts.) Write the complex number in polar form.



16. Given the complex number in polar form $z = 3(\cos 210^\circ + i \sin 210^\circ)$

a. (3 pts.) Write z in rectangular form.

b. (4 pts.) Calculate and simplify: $z^3 =$
(Note: Simplify your answer, but you may leave it in polar form.)

17.

a. (2 pts.) The graph of the equation $4x^2 - 3y^2 - 8x + 6y + 1 = 0$ would be which conic?

b. (2 pts.) The graph of the equation $2x^2 + 2y^2 - 8x + 8y = 0$ would be which conic?

c. (2 pts.) If one of the above is a circle or an ellipse, which one is it? How do you know?

18. If vectors $v = 3i - 5j$ $w = -2i + 3j$.

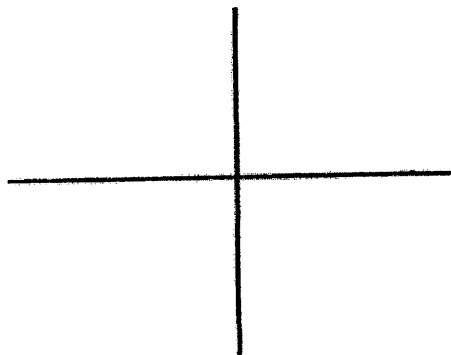
a. (3 pts.) Graphically, show the graph of $v-w$.

b. (2 pts. each) Calculate:

i. $3v - 2w$

ii. The magnitude of vector v .

iii. The unit vector in the same direction as w .



19. (4 pts.) If you graphed the parametric equations $x = 3\cos t$ and $y = 3\sin t$, on the interval $[0, 2\pi]$ the starting point of the graph would be (_____, _____) and the ending point would be (_____, _____).

College Trigonometry

Formula Sheet for Final Exam

Sum and Difference Identities

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

Double Angle Identities

$$\sin 2u = 2 \sin u \cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u = 2 \cos^2 u - 1 = 1 - 2 \sin^2 u$$

Polar Coordinates $(x, y) \leftrightarrow (r, \theta)$

$$x = r \cos \theta$$

$$r = \sqrt{x^2 + y^2}$$

and

$$y = r \sin \theta$$

$$\theta = \tan^{-1} \left(\frac{y}{x} \right)$$

Complex Numbers $x + yi \leftrightarrow r(\cos \theta + i \sin \theta)$

DeMolivre's Theorem

$$\text{If } z = r(\cos \theta + i \sin \theta), \text{ then } z^n = r^n(\cos(n\theta) + i \sin(n\theta))$$

Vectors

If $\mathbf{v} = a_1 \mathbf{i} + b_1 \mathbf{j}$, then:

1. The magnitude of \mathbf{v} : $\|\mathbf{v}\| = \sqrt{a_1^2 + b_1^2}$

2. The unit vector \mathbf{u} in the direction of \mathbf{v} : $\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$

Law of Sines

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

Name: _____

Instructor: _____

20. (3 pts. Each). Write the first 4 terms of the sequences defined as:

a. $\left\{ (-1)^{n-1} \left(\frac{n}{2n-1} \right) \right\}$ _____, _____, _____, _____

b. $a_1 = 3, a_n = \frac{a_{n-1}}{n} + 1$ _____, _____, _____, _____

21. (3 pts.) Write a rule for the nth term of the sequence that begins, $\frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \frac{16}{81}, \dots$

$a_n =$ _____

22. (5 pts.) A wire that is attached to the top of a 75 ft tall television signal transmission tower, makes an angle of 60° with the ground. How long is the wire?

23. (4 pts. Each) Find the indicated part of the triangles below.

a. If $\alpha = 70^\circ$, $\beta = 60^\circ$, and $c=4$, find a.

b. If $a=9$, $b=7$, and $c=10$, find α . (Write your answer in terms of an inverse trig function.)

c. If $a=3$, $b=2$, and $\alpha = 40^\circ$, find c.