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MATH 1120 SPRING 2004 UNIFORM FINAL

Name:____

Circle your section number:

001	002	OL1
Shepherd	Sullivan	Byrne
MW 5:30-6:45	TR 1:00-2:15	N/A

Instructions:

.Put your name on this page and the next page.

.Circle your section number above.

.You are allowed a calculator, pencils, erasers and the note sheet at the back of the test

.If you are unclear what a problem is asking, then talk to your instructor for clarification. You may not ask for hints, verification of formulas, or if you have done the problem correctly. This exam is over what YOU know to date,

.In part 1, just record your answers. In part 2 show your work as partial credit will be given.

.Be neat. If the grader cannot understand what you have recorded, you will not get credit.

DO NOT WRITE BELOW THIS LINE

Part I: page 1. (32)________ Page 2: (18) ________ Part 2: page 2. 26. (8) ________

page 3: 27.(10) ________ 28. (4) ________ 29. (12) ________

page 4: 30. (4) ________ 31. (4) ________ 32. (4) ________

page 5: 33. (6) ________ 34. (6) ________ 35. (10) ________

page 6: 36. (4) ________ 37. (6) ________

page 7: 38. (10) ________ 39. (6) ________ 40. (6) ________ TOTAL: (150)____________

COLLEGE TRIGONOMETRY FINAL EXAM SPRING 2004

NAME:________________________________ SCORE:______________

PART I: Each question is worth 2 points. Just record your final answer. You do not need to show any work and you do not need your calculator. Your answers should be exact.

1. Convert $\frac{5\pi}{4}$ radians to degrees. __________

For problems 2-4 The terminal side of angle θ passes through (-2, 1). Find the exact value of the indicated trigonometric functions of θ.

2. $\sin\theta =$ __________ 3. $\sec\theta =$ __________ 4. $\tan\theta =$ __________

In problems 5-12 find the exact value of each expression.

5. $\cos\left(\frac{11\pi}{6}\right)$ __________

6. $\cot\left(\frac{\pi}{6}\right) =$ __________

7. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) =$ __________radians

8. $\tan^{-1}(-1) =$ __________radians

9. $\tan\pi + \sin\frac{3\pi}{2} =$ __________

10. $\frac{\sec 30^{\circ}}{\csc 60^{\circ}} =$ __________

11. $\tan\left(\sin^{-1}\left(\frac{-2}{5}\right)\right) =$ __________

12. $\cos\left(\cos^{-1}\left(\frac{-\sqrt{5}}{4}\right)\right) =$ __________

13. *If* $\cos\theta < 0$ *and* $\tan\theta > 0$, *then* θ *lies in Quadrant* __________

14. Solve the equation $\sin x = \frac{1}{2}$ on the interval $0 \le x \le 2\pi$. ______________

15. Given the point P with polar coordinates $\left(3, \frac{\pi}{3}\right)$ find another polar coordinates (r, θ) of this same point for which: $r < 0,\ 0 \le \theta \le 2\pi$ ______________

16. Find the rectangular coordinates for $\left(4, -\frac{\pi}{4}\right)$. ______________

17. Write the following complex number in rectangular form (a + bi form).

$3(\cos 210^{\circ} + i\sin 210^{\circ})$ ______________________

In problems 18-21 use the given vectors to answer the questions.

$v = 2i - 5j = \langle 2, -5 \rangle$ *and* $w = -3i + 4j = \langle -3, 4 \rangle$

18. -v + 2w = ______________

19. The magnitude of vector v. __________

20. The dot product $v \bullet w$ **=** __________

21. The unit vector in the same direction as w.

____

In problems 22-24 Identify which conic would be graphed by each equation.

22. $\frac{(x-3)^2}{25} - \frac{(y+2)^2}{36} = 1$ ______________________

23. $4x^2 + 9y^2 = 36$ ______________________

24. $x^2 - 4x = 2y$ ______________________

25. What are the coordinates of the center in problem #22? ______________

<u>Part 2: You must show your work on this part as partial credit will be awarded.</u>

26. (8 pts) Verify the following identities. Show all your steps.

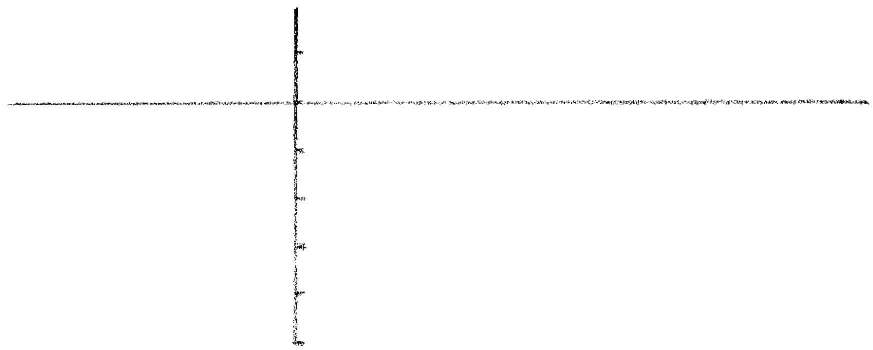
A) $\sin\left(x - \frac{3\pi}{2}\right) = \cos x$

B) $\frac{\cos\alpha}{\cot\alpha} + \frac{\sin\alpha}{\tan\alpha} = \sin\alpha + \cos\alpha$

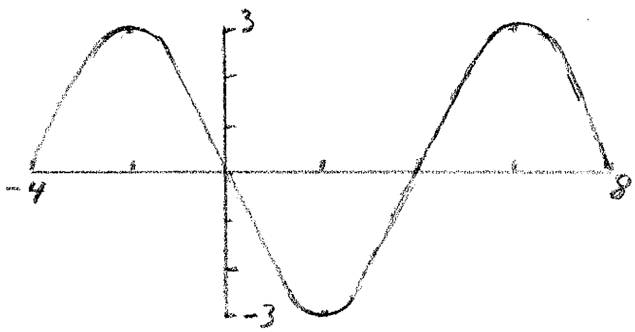
27. (10 pts) Given: $y = 2\cos\left(2x - \frac{\pi}{2}\right) - 3$ **A) amplitude:**__________

B) period: __________ **C) phase shift:**__________ **D) vertical shift:** __________

E) Sketch the graph (at least one period) of the function. Label the important values on the axes.



28. (4 pts) Find an equation for the following graph.



y = ____________________________

29. (12 pts) If $\cos\alpha = \frac{3}{5},\ 0 < \alpha < \frac{\pi}{2},\ and\ \sin\beta = -\frac{5}{13},\ \pi < \beta < \frac{3\pi}{2}$, **find the exact value of:**

A) Sketch α *and* β **in standard position. (2 diagrams)**

B) $\cos(\alpha + \beta)$

C) $\sin(2\beta)$

D) $\tan\frac{\alpha}{2}$

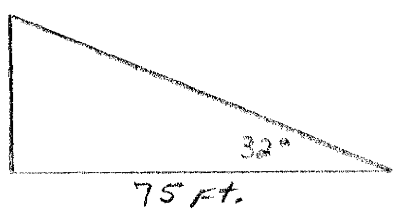
30. (4 pts) Solve the trigonometric equation on the interval: $[0,2\pi)$ (exact values)

$$\cos(2\theta)+3\sin\theta=2$$

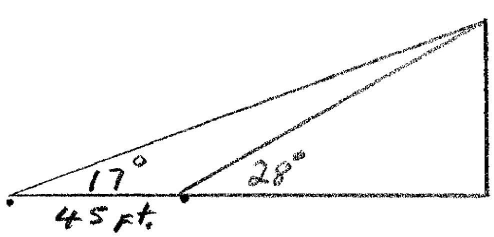
$\theta=$ ____________________

IN PROBLEMS 31 - 33 ROUND ANY DECIMAL ANSWER TO 2 DECIMAL PLACES.

31. (4 pts) From a point on the ground 75 feet from the base of a tower the angle of elevation to the top of the tower is $32°$. Find the height of the tower.



32.(4 pts) Mary needs to determine the height of a tree before cutting it down to be sure that it will not fall on a nearby fence. The angle of elevation of the tree from one position on a flat path from the tree is $28°$ and from a second position 45 feet farther along this path it is $17°$. What is the height of the tree?



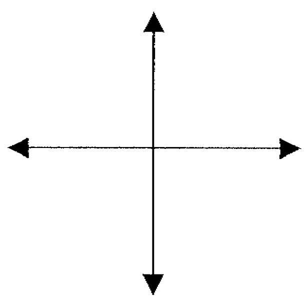
33. (6 pts) Given: a = 12 m, b = 16 m, c = 24 m

A) Find γ

B) Find the area of the triangle.

34. (6 pts) Given the complex number: $-2+3i$

A) Plot the complex number in the complex number plane.



B) Change $-2+3i$ to polar form. (radians) Round answers to 2 decimal places.

35. (10 pts) Given the complex numbers in polar form:

$$z = 6(\cos 100^\circ + i\sin 100^\circ)$$
$$w = 2(\cos 80^\circ + i\sin 80^\circ)$$

A) Find zw in rectangular form (exact values)

B) Find $\frac{z}{w}$ (leave in polar form)

C) Find w^3 in rectangular form (exact values).

36. (4 pts) Two forces of magnitude 40 newtons and 65 newtons act on an object at angles of 40° and 115° respectively. These angles are in standard position. Find the direction and magnitude of the resultant force. Draw and label an appropriate diagram. Round your answers to 2 decimal places.

37. (6 pts) Given the equation: $3x^2 + y^2 - 6x + 8y - 9 = 0$

A) Using the completing the square process find the center of the conic.

B) In order to graph the conic on your calculator, you would first need to solve the equation for y using the quadratic formula. Write the expression you would enter into your calculator to graph the conic.

Center (_____ , _____)

y = ______________________________________

38. (10 pts) Suppose that Robert hit a golf ball with an initial velocity of 145 feet per second at an angle of 28° to the horizontal. Round your answers to 2 decimal places.

A) Find parametric equations that describe the position of the ball as a function of time.

B) How long is the ball in the air?

B) When is the ball at its maximum height and what is the maximum height?

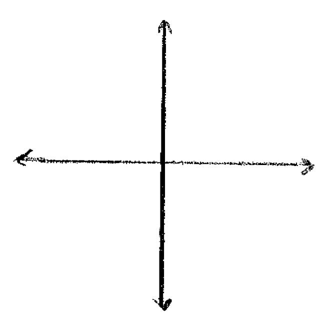
D) Determine the horizontal distance the ball traveled.

39. (6 pts) Write the first 4 terms of the sequences defined as:

A) $a_1 = 4, \quad a_n = 2a_{n-1} - 5$ ____________ , ____________ , ____________ , ____________

B) $\left\{(-1)^n \bullet \dfrac{n+2}{n}\right\}$ ____________ , ____________ , ____________ , ____________

40. (6 pts) Given: $f(x) = \cos^{-1} x$ **A) sketch the graph of f(x)**



B) State the Domain: ____________

C)State the Range: ____________

COLLEGE TRIGONOMETRY NOTE SHEET

CHAPTER 8

1. **Arc Length:** $s = r\theta$ θ *measured in radians* **Velocity:** $v = r\omega$ ω *measured in radians per unit of time*

2. **Cofunctions of complementary angle are equal.**

3. **Remember that some of the trig functions of quadrantal angles are not defined.**

4. Sinusoidal Graphs: $y = A\cos[B(x-C)] + D$ *of* $y = A\sin[B(x-C)] + D$

   |A| amplitude A < 0 Reflection through the x-axis
   Verical stretch or compression

   period= $\dfrac{2\pi}{|B|}$ B < 0 Reflection through the y-axis
   Horizontal stretch or compression. 1/|B|

   C Phase Shift Horizontal shift left or right

   D Vertical shift up or down

5. Steps for fitting data to a Sine function.

   Step 1: Amplitude= $\dfrac{\text{largest data value} - \text{smallest data value}}{2}$ =|A|

   Step 2: Vertical shift= $\dfrac{\text{largest data value} + \text{smallest data value}}{2}$ =D

   Step 3: Determine B. Since the period T, the time it takes for the data to repeat is:
   $T = \dfrac{2\pi}{|B|}$ *therefore* $B = \dfrac{2\pi}{B}$

   Step 4: Determine the horizontal shift of the function by solving the equation
   $y = A\sin[B(x-C)] + D$
   for C by choosing an ordered pair (x, y) from the data. Since answers will vary depending on the ordered pair selected, we will choose the ordered pair for which y is smallest in order to maintain consistency.

Chapter 9 Identities

1. Sum and Difference Formulas: $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

   $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ $\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

2. Double angle Formulas: $\sin 2\theta = 2\sin\theta\cos\theta$; $\cos 2\theta = \cos^2\theta - \sin^2\theta$; $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

3. Half-Angle Formulas: $\sin\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1-\cos\alpha}{2}}$; $\cos\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1+\cos\alpha}{2}}$; $\tan\dfrac{\alpha}{2} = \dfrac{1-\cos\alpha}{\sin\alpha}$

Chapter 10

1. Law of Sines: $\frac{\sin\alpha}{a}=\frac{\sin\beta}{b}=\frac{\sin\gamma}{c}$ 2. Law of Cosines: $a^2=b^2+c^2-2bc\cos\alpha$

3. The area A of a triangle equals one-half the product of two of its sides and the sine of their included angle.

4. Heron's Formula for the area of a triangle. $A=\sqrt{s(s-a)(s-b)(s-c)}$ *where* $s=\frac{1}{2}(a+b+c)$

Chapter 11

1. Polar Coordinates $(x,y)\leftrightarrow(r,\theta)$

$x=r\cos\theta$ $\quad r=\sqrt{x^2+y^2}$

$y=r\sin\theta$ $\quad \theta=\tan^{-1}\frac{y}{x}$ *if* $x>0$

and $\quad \theta=\tan^{-1}\frac{y}{x}+\pi$ *if* $x<0$

Complex Numbers $x+yi\leftrightarrow r(\cos\theta+i\sin\theta)$

De Moivre's Theorem: *If* $z=r(\cos\theta+i\sin\theta)$, *then* $z^n=r^n[\cos(n\theta)+i\sin(n\theta)]$

2. Vectors

if $v=a_1i+b_1j=\langle a_1,b_1\rangle$ *and* $w=a_2i+b_2j=\langle a_1,b_1\rangle$, *then:*

1) The magnitude of v: $\|v\|=\sqrt{a_1^2+b_1^2}$

2) The dot product: $v\bullet w=a_1a_2+b_1b_2$
Two vectors v and w are orthogonal if and only if $v\bullet w=0$

3) The unit vector u in the direction of v: $u=\frac{v}{\|v\|}$

4) Angle θ between u and v: $\cos\theta=\frac{u\bullet v}{\|u\|\,\|v\|}$

5) Writing a vector in terms of magnitude and direction.
$v=\|v\|(\cos\alpha\ i+\sin\alpha\ j)=\langle\|v\|\cos\alpha,\|v\|\sin\alpha\rangle$

Chapter 12

1. Conic Sections

1)Parabola: $(y-k)^2=4a(x-h)$ *or* $(x-h)^2=4a(y-k)$ vertex: (h,k)

2)Ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ *or* $\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$ center: (h,k)

3)Hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$ *or* $\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$ center: (h,k)

4) General form: $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$
$B^2-4AC=0$: parabola $\quad B^2-4AC<0$: ellipise $\quad B^2-4AC>0$: hyperbola

2. Time as a parameter: Projectile motion $x=(v_0\cos\theta)t$ $\quad y=-16t^2+(v_0\sin\theta)t+h$