Math 1120 Uniform Final Exam

May 10, 2003

The time hath come to bid adieu adieu.

Name: _____

Student Number: _____

Circle your section and instructor:

001	002	OL1
N. LeMay	N. LeMay	R. Byrne
Mon/Wed	Tue/Thur	On Line
1:00-2:15	5:30-6:45	

Directions:

1. Complete the section above.
2. You may use your calculator and a 3×5 formula only card.
3. Be sure to **read the instructions for each question carefully** and **clearly mark your answers.**
4. To receive full credit, you must show all your work.
5. If you do not understand any part of the exam, ask me for any clarification.
6. To receive full credit, your work must be legible and neat. Any ambiguities will cost you points.
7. Good Luck and have a wonderful, fullfilling, and safe summer.

Points Part I: _____ **Points Part II:** _____ **Total Points:** _____

You may find the following identities helpful.

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\sin\beta\cos\alpha \qquad \sin(\alpha-\beta)=\sin\alpha\cos\beta-\sin\beta\cos\alpha$$

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta \qquad \cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} \qquad \tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

$$\sin(2\theta)=2\sin\theta\cos\theta \qquad \cos(2\theta)=\begin{cases}\cos^2\theta-\sin^2\theta\\ 2\cos^2\theta-1\\ 1-2\sin^2\theta\end{cases}$$

$$\tan(2\theta)=\frac{2\tan\theta}{1-\tan^2\theta} \qquad \sin\left(\frac{\theta}{2}\right)=\pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\left(\frac{\theta}{2}\right)=\pm\sqrt{\frac{1+\cos\theta}{2}} \qquad \tan\left(\frac{\theta}{2}\right)=\begin{cases}\pm\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}\\ \dfrac{1-\cos\theta}{\sin\theta}\\ \dfrac{\sin\theta}{1+\cos\theta}\end{cases}$$

$$\sin\alpha\sin\beta=\frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right] \qquad \cos\alpha\cos\beta=\frac{1}{2}\left[\cos(\alpha-\beta)+\cos(\alpha+\beta)\right]$$

$$\sin\alpha\cos\beta=\frac{1}{2}\left[\sin(\alpha+\beta)+\sin(\alpha-\beta)\right] \qquad \sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha-\sin\beta=2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2} \qquad \cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha-\cos\beta=-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

Part I: Multiple Choice

Instructions: For the multiple choice questions, place your answer on the provided line. Circled answers will not count. Ambiguous letters are regarded as incorrect responses and no credit will be given. Each question is worth 2 points. Obviously, no partial credit.

1. The $\cos\left(\frac{7\pi}{6}\right)$ is: _____

   A. $-\frac{1}{2}$ B. $\frac{1}{2}$ C. $-\frac{\sqrt{3}}{2}$ D. $\frac{\sqrt{3}}{2}$ E. None of the Above.

2. The complementary angle to $22°$ is: _____

   A. $14°$ B. $68°$ C. $-68°$ D. I don't know. E. None of the above.

3. The positive angle coterminal with $120°$is _____

   A. $480°$ B. $60°$ C. π D. $-180°$ E. None of the above.

4. Given a right triangle with angle C a right angle and angle $A = \frac{\pi}{3}$, then angle B is: _____

   A. $22°$ B. $30°$ C. 0 D. $-\frac{\pi}{6}$ E. None of the above.

5. The period of $f(x) = 2\sin 3x$ is: _____

   A. 2 B. $\frac{2}{\pi}$ C. $\frac{2\pi}{3}$ D. $\frac{3}{2\pi}$ E. None of the above.

6. The amplitude of $f(x) = -2\cos(3x - 1)$ is: _____

   A. -2 B. 2 C. 1 D. None of the above.

7. $15°$ in radians is: _____

   A. $\frac{\pi}{10}$ B. $\frac{10}{\pi}$ C. $-\frac{\pi}{10}$ D. 3.217 E. None of the above.

8. If $\cos\theta < 0$ and $\tan\theta < 0$, then θ lies in quadrant: _____

   A. I B. II C. III D. IV E. Must be polar coordinates.

9. If $\csc x = \frac{9}{5}$ then $\sin x$ is: _____

   A. $-\frac{9}{5}$ B. $\frac{5}{9}$ C. $-\frac{5}{9}$ D. None of the above.

10. If $\sin x = \frac{\sqrt{3}}{2}$, then x could be: _____

    A. $20°$ B. $\frac{\pi}{4}$ C. $\frac{3\pi}{4}$ D.$-\frac{\pi}{2}$ E. None of the above.

11. The range of $\sin x$ is: _____

    A. $(-1, 1)$ B. $(-1, 1]$ C. $[-1, 1)$ D. $[-1, 1]$ E. The same as a tomahawk cruise missle.

12. The polar coordinates $\left(2, \frac{4\pi}{3}\right)$ in rectangular form: _____

    A. $(1, \sqrt{3})$ B. $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ C. $(-1, -\sqrt{3})$ D. $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ E. A square.

13. The polar form of the complex number: $z = 1 + i$ is:
A. $(\sin 45° + i\cos 45°)$ B. $(\cos 45° + i\sin 45°)$ C. $\sqrt{2}(\cos -45° + i\sin -45°)$
D. $\sqrt{2}(\cos 45° + i\sin 45°)$

14. If $\mathbf{w} = 3\mathbf{i} + 4\mathbf{j}$, then $||\mathbf{w}||$ is:
A. 5 B. -5 C. $\frac{1}{5}$ D. $-\frac{1}{5}$ E. None of the above.

15. Let $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{u} = -7\mathbf{i} - 9\mathbf{j}$, then $\mathbf{v} + \mathbf{w}$ is:
A. $\mathbf{0}$ B. $-5\mathbf{i} - 6\mathbf{j}$ C. $5\mathbf{i} + 6\mathbf{j}$ D. $-5\mathbf{i} + 6\mathbf{j}$ E. $5\mathbf{i} - 6\mathbf{j}$

16. Let $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{u} = -7\mathbf{i} - 9\mathbf{j}$, then $\mathbf{v} \cdot \mathbf{u}$ is:
A. 41 B. 14 C. -41 D. -14 E. What does this mean?

17. $4\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$ in standard $a + bi$ form is:
A. $2\sqrt{2} + 2\sqrt{2}i$ B. $-2\sqrt{2} + 2\sqrt{2}i$ C. $-2\sqrt{2} - 2\sqrt{2}i$ D. $2\sqrt{2} - 2\sqrt{2}i$
E. None of the above.

18. The vector $\mathbf{v}$ is represented by the directed line segment $\overrightarrow{PQ}$. Write $\mathbf{v}$ in the form $a\mathbf{i} + b\mathbf{i}$, where $P = (-3, 1)$ and $Q = (4, -2)$
A. $\langle 1, 1\rangle$ B. $\langle -7, -3\rangle$ C. $\langle 3, 7\rangle$ D. $\langle 7, -3\rangle$ E. $\langle -7, 3\rangle$

19. Are the vectors $\mathbf{v} = -4\mathbf{i} + 2\mathbf{j}$ and $\mathbf{u} = 2\mathbf{i} - 4\mathbf{j}$ orthogonal?
A. No B. Yes C. I Don't know. D. None of the above.

20. The domain for $g(x) = \cos^{-1} x$ is:
A. $[0, \pi]$ B. $(-1, 1)$ C. $[-1, 1]$ D. $(0, \pi)$ E. None of the above.

21. The angle 700° is in quadrant:
A. I B. II C. III D. IV E. Quadrant Mojo

22. Are the vectors $\mathbf{v} = 5\mathbf{i} - 2\mathbf{j}$ and $\mathbf{u} = -10\mathbf{i} + 4\mathbf{j}$ parallel?
A. I don't know what parallel means. B. No C. I don't know D. Yes

23. Find the length of the arc subtended by an angle of 30° on a circle of radius 2.
A. 60 B. 1 C. $\frac{2\pi}{3}$ D. π E. $\frac{\pi}{3}$

24. Find the area of the sector given the angle of 60° and a radius of 3.
A. π B. 3π C. 2π D. 180 E. 540

25. Given a right triangle Δabc, where c is the hypotenuse, are the following relations:
$\sin\theta = \frac{\text{opp}}{\text{hyp}}$, $\cos\theta = \frac{\text{hyp}}{\text{adj}}$, $\tan\theta = \frac{\text{opp}}{\text{adj}}$ correct?
A. Yes B. No C. I don't know. D. None of the above.

Part II: Computational Questions

Directions: Read each question carefully. If you do not understand the question and need clarification, **ask me.** Clearly mark your final answer. Write legibly, clearly, and neatly. Any ambiguity in your solution, no credit will be given.

1. (10 points) Given $\sin\theta = \frac{12}{13}$ and θ is in Quad. II, compute the exact values of the following trig functions:

   (a) $\cos\theta$

   (b) $\tan\theta$

   (c) $\sec\theta$

   (d) $\csc\theta$

   (e) $\cot\theta$

2. (10 points) Let $f(x) = 2\cos\left(\frac{x}{2} + \frac{\pi}{2}\right)$ determine the following:

   (a) Amplitude

   (b) Period:

   (c) Phase shift:

   (d) Vertical Shift:

   (e) Range:

3. (5 points) Verify the identity $\cos(2x) = \cos^4 x - \sin^4 x$

4. (5 points) Find the exact value of $\sin\left(-\frac{\pi}{12}\right)$

5. (5 points) Solve the equation for the exact values of x in the interval $0 \leq x < 2\pi$, $4\sin^2 x = 1 + 4\cos x$

6. Given $\sin\alpha = \frac{4}{5}$, $0 < \alpha < \frac{\pi}{2}$ and $\sin\beta = \frac{5}{13}$, $\frac{\pi}{2} < \beta < \pi$ Compute the exact values of the following:

   (a) (5 points) $\cos(\alpha + \beta)$

   (b) (5 points) $\sin\left(\frac{\beta}{2}\right)$

7. (5 points) A hot air balloon is between two major cities. The pilot determines the angle of depression to city A is 50° and the distance from the balloon to the city is 20 miles. The pilot also determines the angle of depression to city B is 30° and the distance from the balloon to the city is 50 miles. How far apart are the cities?

8. (5 points) Jessie is standing 40 feet from a tree and measures the angle of elevation from the gound to the top of the tree to be 55°. How tall is the tree?

9. (5 points) Let $z = 2(\cos\pi + i\sin\pi)$, compute z^5 Leave your final answer in trig / polar form.

10. (5 points) Let $z = 4(\cos\pi + i\sin\pi)$ and $w = 2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$, compute zw and leave your answer in trig / polar form.