College Trigonometry **Final Exam** **Fall 2006**

Name ____

1. (2 pts) Convert the angle whose measure is 140° to radians. Simplify your answer. ____________ radians

2. (6 pts) The terminal side of angle θ passes through (-4 , -3). Find the exact value of the indicated trigonometric functions of θ.

$\sin\theta =$ ____________ $\cot\theta =$ ____________ $\sec\theta =$ ____________

3. (2 pts each) Find the exact value of each expression. Circle your final answer.

a. $\cos\left(\frac{4\pi}{3}\right) =$

d. $\csc\left(\frac{7\pi}{4}\right) =$

b. $\sin\left(\frac{7\pi}{2}\right) =$

e. $\cos^{-1}(0) =$

c. $\tan\left(\frac{5\pi}{6}\right) =$

f. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) =$

4. (2 pts) If $\cos\theta < 0$ and $\tan\theta < 0$, then θ lies in Quadrant ________

5. (5 pts) Sketch the graph of **$y = \sec x$.** State the **domain** and **range**.

Domain: ______________________________

Range: ______________________________

6. (7 pts each) Sketch the graph (at least one period) of each function below and state the amplitude, period, vertical shift, and phase shift. Label the important values on the axes.

a. $y = -\cos x + 1$

Amplitude: ___________ Period: _____________

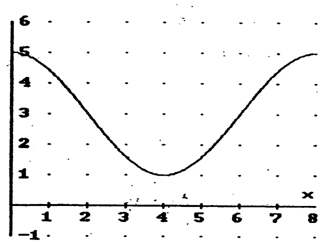
Vertical shift: ________________ Phase shift: _________________

b. $y = 4\sin(3x - \pi)$

Amplitude: ___________ Period: _____________

Vertical shift: ________________ Phase shift: _________________

7. (5 pts) Write an equation for the function shown below.



y = ______________________________

8. (4 pts each) Prove the following identities. Show all steps.

a. $\cos\left(\theta - \frac{3\pi}{2}\right) = -\sin\theta$

b. $\tan\theta + \cot\theta = \sec\theta \cdot \csc\theta$

9. (4 pts) Find the exact value of $\sin\left(\frac{11\pi}{12}\right)$.

10. (4 pts each) If $\sin\alpha = \frac{3}{5}$, $\frac{\pi}{2} < \alpha < \pi$, and $\cos\beta = \frac{-5}{13}$, $\frac{\pi}{2} < \beta < \pi$, find the exact value of:

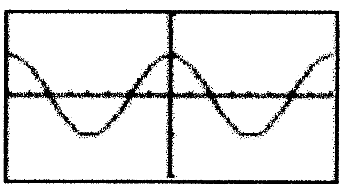
a. $\sin(\alpha + \beta) =$

b. $\cos(2\beta) =$

11. (5 pts) Solve the following trigonometric equation on the interval $[\,0\,,\,2\pi\,]$. Show your work.

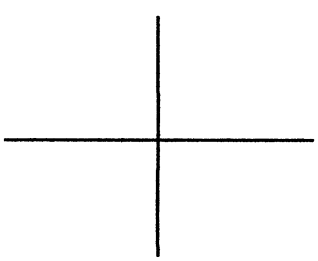
$2\sin^2\theta - \sin\theta = 0$ $\theta =$ ______________________________

12. The graph of **f(x) = cosx** on $[\,-2\pi\,,\,2\pi\,]$ is shown below.



a. (2 pts) The function **f** is not a one-to-one function. To make it a one-to-one function, restrict the domain to [________ , __________].

b. (5 pts) The inverse of **f** is $\mathbf{f^{-1}(x) = \cos^{-1}x}$. Sketch the graph of $\mathbf{f^{-1}}$ and state the **domain** and **range**.



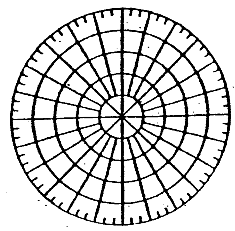
Domain: ____________________

Range: ____________________

c. (2 pts) Find: $\mathbf{\cos^{-1}\left(-\frac{1}{2}\right)} =$ ____________

13. Given the point $\left(-4, \frac{5\pi}{4}\right)$ in polar coordinates,

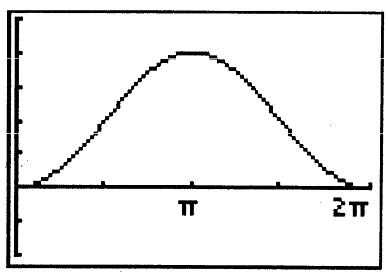
a. (2 pts) Plot the point.

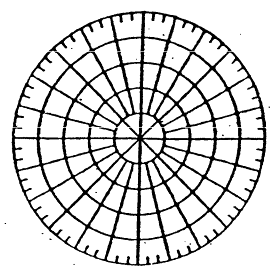


b. (2 pts) Find other polar coordinates (r , θ) of the point for which $r > 0$ and $0 < \theta < 2\pi$ (_____ , _____)

c. (2 pts) Find the rectangular coordinates of the point. (________ , ________)

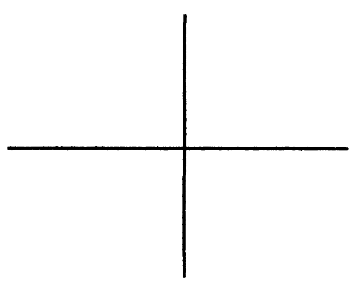
14. (4 pts) Given the polar equation $r = 2 - 2\cos\theta$, and its corresponding graph in rectangular form, sketch a graph of the equation in polar equations.





15. Given the complex number 0 - 5i,

a. (1 pt) Plot the complex number in the complex number plane.



b. (3 pts) Write the complex number in polar form.

16. Given the complex number in polar form $z = 2(\cos 120^\circ + i \sin 120^\circ)$,

a. (3 pts) Write z in rectangular form.

b. (4 pts) Calculate and simplify: $z^5 =$
(Note: Simplify your answer , but you may leave it in polar form.)

17. Given the equation $2x^2+3y^2+12x-24y+60=0$

a. (2 pts) The graph of the equation would be which conic? ______________________________

b. (4 pts) In order to graph the conic on your calculator, you would first need to solve the equation for y using the Quadratic formula. Write the expressions you would enter into your calculator to graph the conic.

Y = ____

18. If vectors **v** = 3**i** - **j** and w = 2**i** + 5**j**,

a. (3 pts) Graphically, show the graph of **v** + **w**.

b. (2 pts each) Calculate:

i. 2**v** - 3**w** =

ii. The magnitude of vector **w**.

iii. The unit vector in the same direction as **v**.

19. (4 pts) If you graphed the parametric equations $x=\frac{12}{t}$ and $y=4-t^2$, for the values of t in the interval [-3 , 6], the starting point of the graph would be (_______ , ________) and the ending point of the graph would be (_______ , ________) .

20. (4 pts each) Write the first 4 terms of the sequences defined as:

a. $\left\{(-1)^n \cdot \frac{n^2}{n+5}\right\}$. __________ , __________ , __________ , __________

b. $a_1 = -5, \quad a_n = n + a_{n-1}$ __________ , __________ , __________ , __________

21. (3 pts) Write a rule for the nth term of the sequence that begins $\frac{1}{3}, \frac{3}{6}, \frac{5}{9}, \frac{7}{12}, \ldots$

$a_n =$ ____________________

22. (5 pts) Write the first two terms and the tenth term of the series. Then, using the properties of sigma notation, find the sum of the series.

$\sum_{k=1}^{10} k^2 - 2k + 2 =$ _____ + _____ + ... + _____ =

23. (4 pts) The angle of elevation to the top of a building from a point 300 ft away from the base of the building on level ground is $30°$. Find the height of the building.

24. (5 pts each) Find the indicated part of the triangles below.

a. If $\alpha = 60°$, $b = 6$, and $c = 8$, find a .

b. If $a = 12$, $b = 4$, and $\alpha = 120°$, find β. (Write your answer in terms of an inverse trig function)

College Trigonmetry **Formula Sheet for Final Exam**

Sum and Difference Identities

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$
$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

Double Angle Identities

$$\sin 2u = 2 \sin u \cos u$$
$$\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 - 1 = 1 - 2\sin^2 u$$

Polar Coordinates $(x, y) \leftrightarrow (r, \theta)$ $x = r\cos\theta$ $r = \sqrt{x^2 + y^2}$

and $y = r\sin\theta$ $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

Complex Numbers $x + yi \leftrightarrow r(\cos\theta + i\sin\theta)$

DeMoivre's Theorem

If $z = r(\cos\theta + i\sin\theta)$, then $z^n = r^n(\cos(n\theta) + i\sin(n\theta))$

Vectors

If $\mathbf{v} = a_1\mathbf{i} + b_1\mathbf{j}$

1. The magnitude of $\mathbf{v}$: $\|\mathbf{v}\| = \sqrt{a_1^2 + b_1^2}$
2. The unit vector $\mathbf{u}$ in the direction of $\mathbf{v}$: $\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$

Law of Sines

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

Properties of Series

1. $\sum_{k=1}^{n} c = c \cdot n$
2. $\sum_{k=1}^{n} c \cdot a_k = c \cdot \sum_{k=1}^{n} a_k$
3. $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$
4. $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$