College Trigonometry **Final Exam** **Fall 2005**

Name _____

1. (2 pts) Convert the angle whose measure is 80° to radians. Simplify your answer. _____ radians

2. (6 pts) The terminal side of angle θ passes through (6 , -8). Find the exact value of the indicated trigonometric functions of θ. Simplify your answers.

$\cos\theta =$ _____ $\tan\theta =$ _____ $\csc\theta =$ _____

3. (2 pts each) Find the exact value of each expression. Circle your final answer.

a. $\sin\left(\frac{5\pi}{6}\right) =$

b. $\cos\left(\frac{5\pi}{2}\right) =$

c. $\tan\left(\frac{\pi}{3}\right) =$

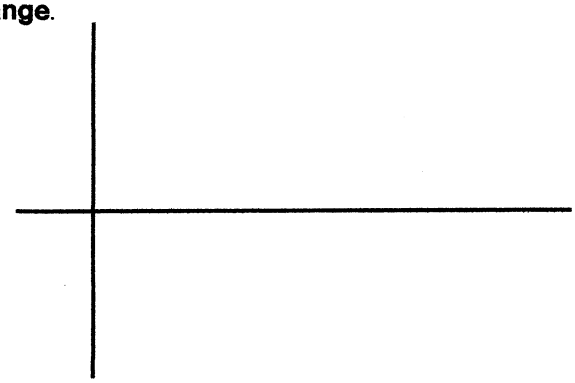
d. $\sec\left(\frac{5\pi}{4}\right) =$

e. $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) =$

f. $\tan^{-1}(1) =$

4. (2 pts) If $\sin\theta < 0$ and $\tan\theta > 0$, then θ lies in Quadrant _____

5. (5 pts) Sketch the graph of **y = cotx** on [0 , 2π]. Show asymptotes with dotted lines. State the **domain** and **range**.

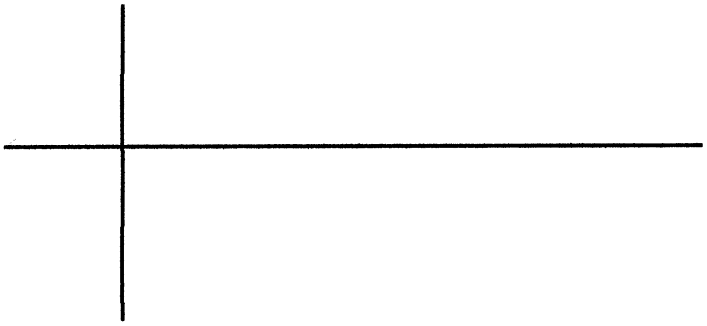


Domain: _____

Range: _____

6. (7 pts each) Sketch the graph (at least one period) of each function below and state the amplitude, period, vertical shift, and phase shift. Label the important values on the axes.

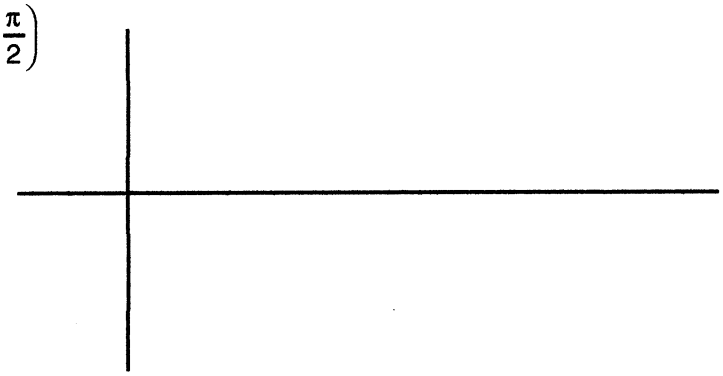
a. y = 2sinx - 1



Amplitude: _____ Period: _____

Vertical shift: _____ Phase shift: _____

b. $y = -\cos\left(2x + \frac{\pi}{2}\right)$

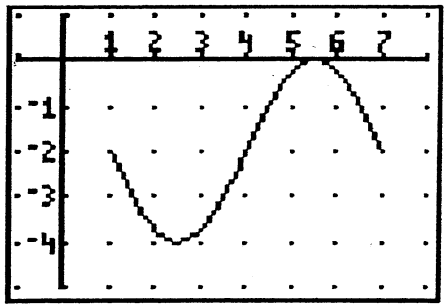


Amplitude: _____ Period: _____

Vertical shift: _____ Phase shift: _____

7. (5 pts) One period of a trig function is shown below. Write an equation for the function.

y = _____



8. (4 pts each) Prove the following identities. Show all steps.

a. $\sin\left(\frac{3\pi}{2}+\theta\right)=-\cos\theta$

b. $\frac{\sec^2\theta-1}{\sec^2\theta}=\sin^2\theta$

9. (4 pts) Find the exact value of $\cos\left(\frac{5\pi}{12}\right)$.

10. (4 pts each) If $\sin\alpha = \frac{3}{5}$, $0 < \alpha < \frac{\pi}{2}$, and $\cos\beta = \frac{12}{13}$, $\frac{3\pi}{2} < \beta < 2\pi$, find the exact value of:

a. $\cos(\alpha - \beta) =$

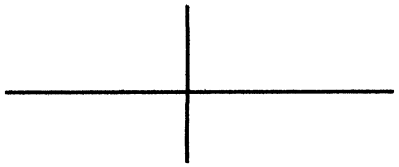
b. $\sin(2\alpha) =$

11. (5 pts) Solve the following trigonometric equation on the interval [0 , 2π]. Show your work.

$\sin 2\theta + \cos\theta = 0$ $\theta =$ _____

12. (1 pt) Sketch the graph of **f(x) = sinx** on [-2π , 2π].

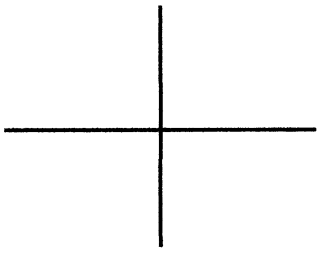
a.



b. (2 pts) The function **f** is not a one-to-one function. To make it a one-to-one function, restrict the domain to:

[_____ , _____].

c. (4 pts) The inverse of **f** is $\mathbf{f^{-1}(x) = \sin^{-1}x}$. Sketch the graph of $\mathbf{f^{-1}}$ and state the **domain** and **range**.

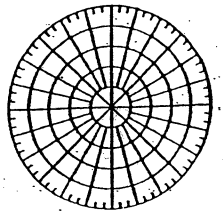


Domain: _____

Range: _____

d. (2 pts) Find: $\sin^{-1}(-1) =$ _____

13. Given the point $\left(-2, \frac{5\pi}{3}\right)$ in polar coordinates,

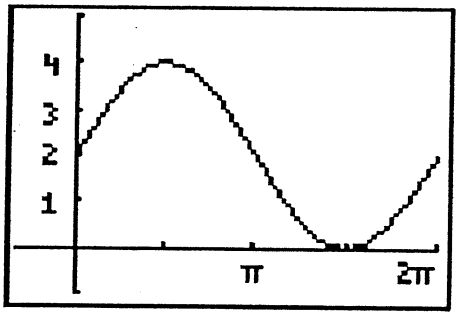


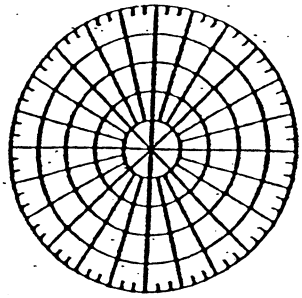
a. (2 pts) Plot the point.

b. (2 pts) Find other polar coordinates (r , θ) of the point for which $r > 0$ and $0 < \theta < 2\pi$ (_____ , __

c. (2 pts) Find the rectangular coordinates of the point. (_____ , _____)

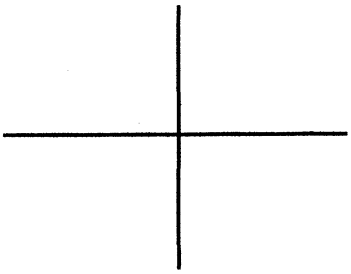
14. (4 pts) Given the polar equation $r = 2 + 2\sin\theta$ and its corresponding graph in rectangular form. Sketch a graph of the equation in polar coordinates.





15. Given the complex number -6 + 6i,

a. (1 pt) Plot the complex number in the complex number plane.



b. (3 pts) Write the complex number in polar form.

16. Given the complex number in polar form $z = 4(\cos 150° + i\sin 150°)$,

a. (3 pts) Write z in rectangular form.

b. (4 pts) Calculate and simplify: z^3 =
(Note: Simplify your answer, but you may leave it in polar form.)

17. Given the equation $x^2 - 4y^2 + 16y - 20 = 0$

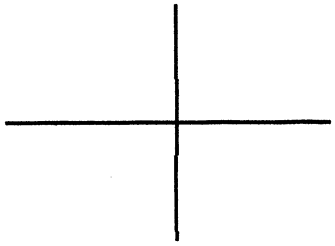
a. (2 pts) The graph of the equation would be which conic? _____

b. (4 pts) In order to graph the conic on your calculator, you would first need to solve the equation for y using the Quadratic formula. Write the expressions you would enter into your calculator to graph the conic.

Y = _____

18. If vectors **v** = 2**i** + 4**j** and w = 4**i** - 3**j**,

a. (3 pts) Graphically, show the graph of **v** - **w**.



b. (2 pts each) Calculate:

i. 2**v** + 3**w** =

ii. The magnitude of vector **v**.

iii. The unit vector in the same direction as **w**.

19. (4 pts) If you graphed the parametric equations $x = \sqrt{t} + 4$ and $y = -4t^2$, on the interval [1 , 4], the starting point of the graph would be (_____ , _____) and the ending point would be (_____ , _____).

20. (4 pts each) Write the first 4 terms of the sequences defined as:

a. $\left\{(-1)^{n+1} \cdot \frac{n}{n+3}\right\}$ _____ , _____ , _____ , _____

b. $a_1 = -2, \quad a_n = n \bullet a_{n-1}$ _____ , _____ , _____ , _____

21. (3 pts) Write a rule for the nth term of the sequence that begins $\frac{2}{3}, \frac{4}{9}, \frac{6}{27}, \frac{8}{81}\ldots\ldots$

$a_n =$ _____

22. (5 pts) Write the first two terms and the twelfth term of the series. Then, using the properties of sigma notation, find the sum of the series.

$$\sum_{k=1}^{12} k^2 + 3k - 6 = \_\_\_\_\_ + \_\_\_\_\_ + \ldots + \_\_\_\_\_ =$$

23. (4 pts) A wire that is attached to the top of an 80 ft tall radio transmission tower, makes an angle of 45° with the ground. How long is the wire?

24. (5 pts each) Find the indicated part of the triangles below.

a. If $\alpha = 30°$, $\beta = 105°$, and $a = 7$, find c.

b. If $a = 5$, $b = 8$, and $c = 9$, find α. (Write your answer in terms of an inverse trig function)

College Trigonmetry **Formula Sheet for Final Exam**

Sum and Difference Identities

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$
$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

Double Angle Identities

$$\sin 2u = 2 \sin u \cos u$$
$$\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 - 1 = 1 - 2\sin^2 u$$

Polar Coordinates $(x, y) \leftrightarrow (r, \theta)$ $\quad x = r\cos\theta \quad r = \sqrt{x^2 + y^2}$

and $\quad y = r\sin\theta \quad \theta = \tan^{-1}\left(\frac{y}{x}\right)$

Complex Numbers $x + yi \leftrightarrow r(\cos\theta + i\sin\theta)$

DeMoivre's Theorem

If $z = r(\cos\theta + i\sin\theta)$, then $z^n = r^n(\cos(n\theta) + i\sin(n\theta))$

Vectors

If $\mathbf{v} = a_1\mathbf{i} + b_1\mathbf{j}$, then:

1. The magnitude of $\mathbf{v}$: $\|\mathbf{v}\| = \sqrt{a_1^2 + b_1^2}$
2. The unit vector $\mathbf{u}$ in the direction of $\mathbf{v}$: $\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$

Law of Sines

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

Properties of Series

1. $\sum_{k=1}^{n} c = c \cdot n$
2. $\sum_{k=1}^{n} c \cdot a_k = c \cdot \sum_{k=1}^{n} a_k$
3. $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$
4. $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$