# MATH 1120 UNIFORM FINAL EXAM

## December 7,2002

Name: _____

Student Number: _____

Circle your section and instructor:

| 001 | 002 | OL1 |
|---|---|---|
| B. MacMillan<br>Mon/Wed<br>1:00-2:15 | J.Bartuska<br>Tue/Thur<br>5:30-6:45 | R. Byrne<br>On Line |

Directions:

1. Complete the section above.
2. Put your name on ***Page 1*** of the test. You should have **7** pages of this test, plus the Formula Sheet.
3. Show all work and be neat! If we can not follow your work, you will not receive any credit.
4. If you are confused about what a problem is asking, ask your instructor. You may <u>not</u> ask for hints or a verification on how you have completed a problem.
5. You are <u>not</u> allowed calculators or computers.

**DO NOT WRITE IN THIS SPACE**

| | | | |
|---|---|---|---|
| Page 1. (23 pts) | Page 2. (20 pts) | Page 3. (17 pts) | Page 4. (22 pts) |
| Page 5. (18 pts) | Page 6. (21 pts) | Page 7. (29 pts) | |

TOTAL: _____ (out of 150 points)

**College Trigonometry** **Final Exam** **Fall 2002**

**Name** _____

1. (3 pts) Convert the angle whose measure is 140° to radians. Simplify your answer. _____ radians

2. (6 pts) The terminal side of angle $\theta$ passes through ( -4 , -3 ). Find the exact value of each of the six trigonometric functions of $\theta$.

$\sin\theta =$ _____ $\cot\theta =$ _____

$\cos\theta =$ _____ $\sec\theta =$ _____

$\tan\theta =$ _____ $\csc\theta =$ _____

3. (2 pts each) Find the exact value of each expression. Circle your final answer.

a. $\cos\left(\frac{4\pi}{3}\right) =$

b. $\sin\left(\frac{7\pi}{2}\right) =$

c. $\tan\left(\frac{5\pi}{6}\right) =$

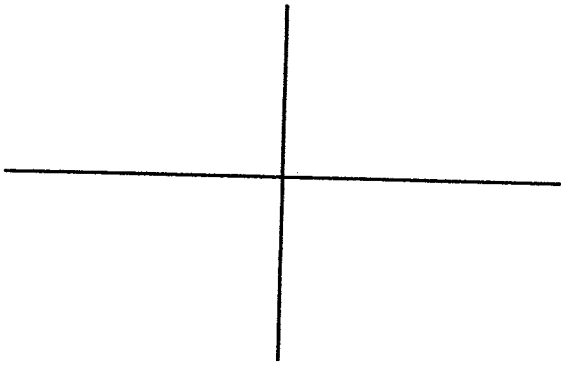
d. $\csc\left(\frac{7\pi}{4}\right) =$

e. $\cos^{-1}(0) =$

f. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) =$

4. (2 pts) If $\cos\theta < 0$ and $\tan\theta < 0$, then $\theta$ lies in Quadrant _____

5. (6 pts) Sketch the graph of **y = secx.** State the **domain** and **range**.

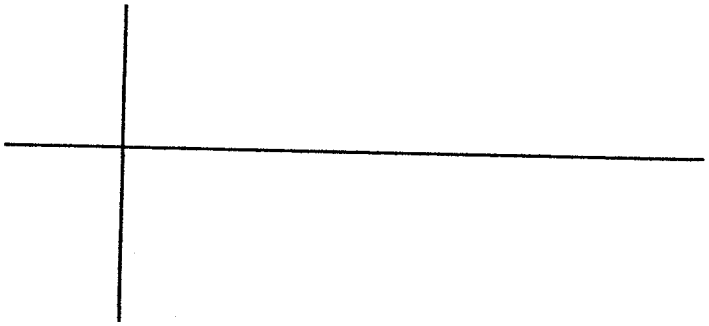


Domain: _____

Range: _____

6. (7 pts each) Sketch the graph (at least one period ) of each function below and state the amplitude, period, vertical shift, and phase shift. Label the important values on the axes.

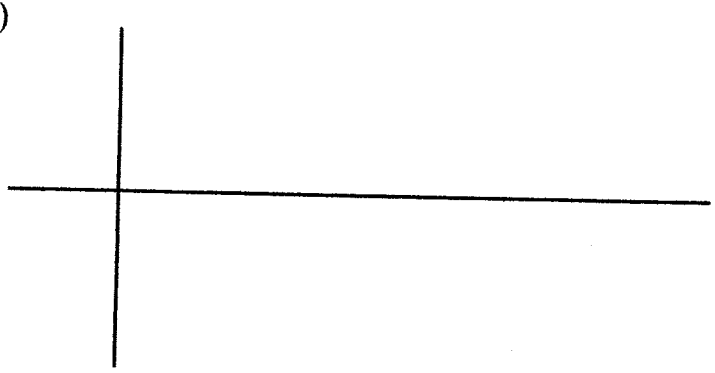
a. $y = -\cos x + 1$



Amplitude: _____ Period: _____

Vertical shift: _____ Phase shift: _____

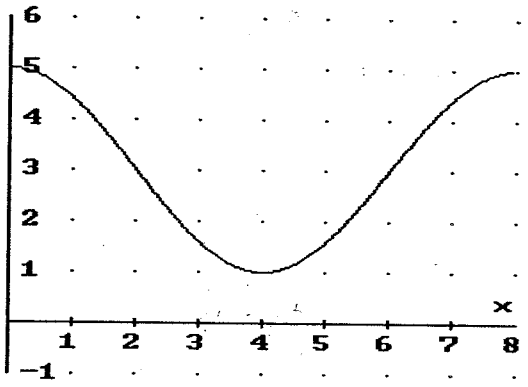
b. $y = 4\sin(3x - \pi)$



Amplitude: _____ Period: _____

Vertical shift: _____ Phase shift: _____

7. (5 pts) Write an equation for the function shown below.




y = _____

8. (4 pts each) Prove the following identities. Show all steps.

a. $\cos\left(\theta - \frac{3\pi}{2}\right) = -\sin\theta$

b. $\tan\theta + \cot\theta = \sec\theta \cdot \csc\theta$

9. (4 pts) Find the exact value of $\sin\left(\frac{11\pi}{12}\right)$.

10. (4 pts each) If $\sin\alpha = \frac{3}{5}$, $\frac{\pi}{2} < \alpha < \pi$, and $\cos\beta = \frac{-5}{13}$, $\frac{\pi}{2} < \beta < \pi$, find the exact value of:

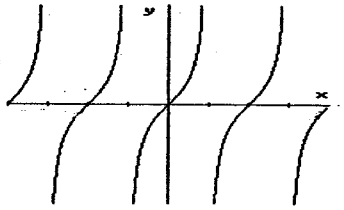
a. $\sin(\alpha+\beta) =$

b. $\cos(2\beta) =$

11. (5 pts) Solve the following trigonometric equation on the interval $[\,0\,,\,2\pi\,]$. Show your work.

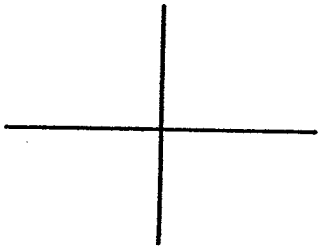
$2\sin^2\theta - \sin\theta = 0$ $\theta =$ _____

12. Below is the graph of $\mathbf{f(x) = \tan x}$ on $[\,-2\pi\,,\,2\pi\,]$.



a. (2 pts) The function **f** is not a one-to-one function. To make it a one-to-one function, restrict the domain to [ _____ , _____ ].

b. (5 pts) The inverse of **f** is $\mathbf{f^{-1}(x) = \tan^{-1} x}$. Sketch the graph of $\mathbf{f^{-1}}$ and state the **domain** and **range**.



Domain: _____

Range: _____

c. (2 pts) Find: $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) =$ _____

13. Given the point $\left(-4, \frac{5\pi}{4}\right)$ in polar coordinates,

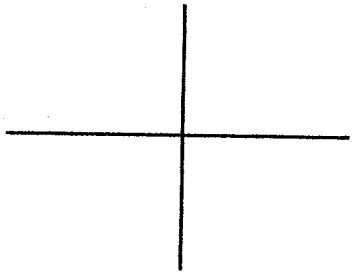
a. (2 pts) Plot the point.

b. (2 pts) Find other polar coordinates ( r , θ ) of the point for which $r > 0$ and $0 < \theta < 2\pi$ ( _____ , _____ )

c. (3 pts) Find the rectangular coordinates of the point. ( _____ , _____ )

14. Given the complex number 0 - 5i,

a. (1 pt) Plot the complex number in the complex number plane.



b. (3 pts) Write the complex number in polar form.

15. Given the complex number in polar form $z = 2(\cos 120° + i \sin 120°)$,

a. (3 pts) Write z in rectangular form.

b. (4 pts) Calculate and simplify: $z^5 =$

16. Given the equation $2x^2 - 3y^2 + 12x - 24y + 60 = 0$

a. (2 pts) The graph of the equation would be which conic? _____

b. (4 pts) In order to graph the conic on your calculator, you would first need to solve the equation for y using the Quadratic formula. Write the expressions you would enter into your calculator to graph the conic.

Y = _____

17. If vectors $\mathbf{v} = 3\mathbf{i} - \mathbf{j}$ and $\mathbf{w} = 2\mathbf{i} + 5\mathbf{j}$,

a. (3 pts) Graphically, show the graph of $\mathbf{v} + \mathbf{w}$.

b. (2 pts each) Calculate:

i. $2\mathbf{v} - 3\mathbf{w} =$

ii. The magnitude of vector $\mathbf{w}$.

iii. The dot product $\mathbf{v} \bullet \mathbf{w}$.

iv. The unit vector in the same direction as $\mathbf{v}$.

18. (4 pts) If you graphed the parametric equations $x = \frac{12}{t}$ and $y = 4 - t^2$, on the interval [ -3 , 6 ], the starting point of the graph would be ( _____ , _____ ) and the ending point would be ( _____ , _____ ).

19. (4 pts each) Write the first 4 terms of the sequences defined as:

a. $\left\{(-1)^n \cdot \frac{n^2}{n+5}\right\}$ _____, _____, _____, _____

b. $a_1 = -5, \quad a_n = n + a_{n-1}$ _____, _____, _____, _____

20. (3 pts) Write a rule for the nth term of the sequence that begins $\frac{1}{3}, \frac{3}{6}, \frac{5}{9}, \frac{7}{12}, \ldots$

$a_n =$ _____

21. (3 pts) Write the expression you would use to find the sum of the first 15 terms of the geometric series that begins: $-2+6-18+54-\ldots$ (Note: You do not need to simplify the expression.)

$S_{15} =$ _____

22. (5 pts) The angle of elevation to the top of a building from a point 300 ft away from the base of the building on level ground is 30°. Find the height of the building.

23. (5 pts each) Find the indicated part of the triangles below.

a. If $\alpha = 60°$, $b = 6$, and $c = 8$, find $a$.

b. If $a = 12$, $b = 4$, and $\alpha = 120°$, find $\beta$. (Write your answer in terms of an inverse trig function)

# College Trigonmetry

# Formula Sheet for Final Exam

## Sum and Difference Identities

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$
$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

## Double Angle Identities

$$\sin 2u = 2 \sin u \cos u$$
$$\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 - 1 = 1 - 2\sin^2 u$$

**Polar Coordinates** $(x,y) \leftrightarrow (r,\theta)$ $\quad x = r\cos\theta \quad r = \sqrt{x^2 + y^2}$

**and** $\quad y = r\sin\theta \quad \theta = \tan^{-1}\left(\frac{y}{x}\right)$

**Complex Numbers** $x + yi \leftrightarrow r(\cos\theta + i\sin\theta)$

## Vectors

If $\mathbf{v} = a_1\mathbf{i} + b_1\mathbf{j}$ and $\mathbf{w} = a_2\mathbf{i} + b_2\mathbf{j}$, then:

1. The magnitude of $\mathbf{v}$: $\|\mathbf{v}\| = \sqrt{a_1^2 + b_1^2}$
2. The dot product: $\mathbf{u} \bullet \mathbf{v} = a_1a_2 + b_1b_2$
3. The unit vector $\mathbf{u}$ in the direction of $\mathbf{v}$ : $\mathbf{u} = \dfrac{\mathbf{v}}{\|\mathbf{v}\|}$
4. Angle $\theta$ between $\mathbf{u}$ and $\mathbf{v}$: $\cos\theta = \dfrac{\mathbf{u} \bullet \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|}$

## Law of Sines

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

## Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

## Geometric Sequences and Series

nth term: $a_n = ar^{n-1}, r \neq 0$

Sum of the first n terms: $S_n = a\dfrac{1-r^n}{1-r}$ , $r \neq 0,1$