Math 1080 Uniform Final Exam

May 6, 2006

Print your name: _____

Circle your section below.

Batista	Kurtz	Lana	Lana
001	002	003	004
MW 10:00 AM	MW 1:00 PM	T/R 10:00 AM	T/R 5:30 PM

Instructions:

- Put your name on this page and on the next page.
- Circle your section number above.
- The exam is closed notes and closed book. You may NOT use a calculator at any time during the exam. A sheet of formulas is provided at the end of the exam.
- Answer each question in the space provided. Show your work when possible. Partial credit is possible on some questions, but only if you show your work.
- You may not communicate with anyone other than the instructor during the exam. The instructor may help to clarify the meaning of a question but is not allowed to give you hints, verify formulas, or tell you if you have done the problem correctly.
- Be neat. If the grader cannot understand what you have written, you will not receive credit.

DO NOT WRITE BELOW THIS LINE

Page 1 (10 pts) _____

Page 2 (17 pts) _____

Page 3 (15 pts) _____

Page 4 (15 pts) _____

Page 5 (10 pts) _____

Page 6 (9 pts) _____

Page 7 (13 pts) _____

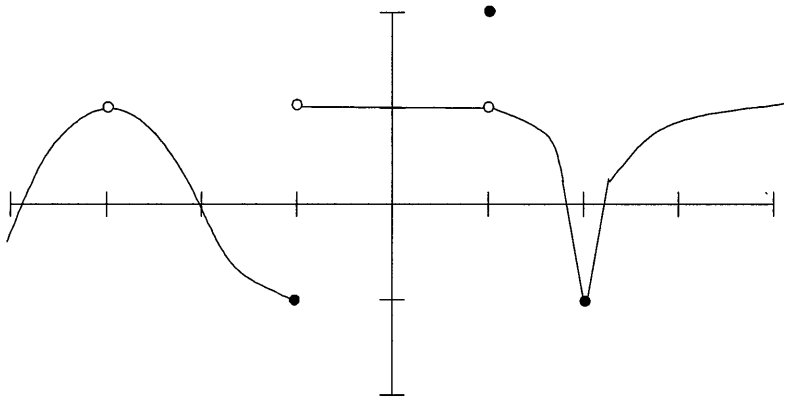
Page 8 (15 pts) _____

Page 9 (15 pts) _____

Page 10 (6 pts) _____

Total (125 pts) _____

1) (10 pts) Consider the graph of $y = f(x)$ given below. Find each of the following if possible.



a) $\lim_{x \to 1} f(x)$ _____

b) $\lim_{x \to -1} f(x)$ _____

c) $\lim_{x \to -1^+} f(x)$ _____

d) $f(1)$ _____

e) $f(-1)$ _____

f) $f'(0)$ _____

g) $f'(2)$ _____

h) $f'(-3)$ _____

i) Is $f(x)$ continuous at x = 2? _____

j) Is $f(x)$ continuous at $x = 1$? _____

2) (9 pts) Evaluate each limit. If it does not exist, write DNE.

a) $\lim_{x\to -3} \dfrac{x^2-9}{x+3}$

b) $\lim_{x\to 2} \dfrac{6}{x-2}$

c) $\lim_{x\to 2} \left(x^2+7x\right)$

3) (8 pts) Use the limit definition of the derivative, $f'(x)=\lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$, and the four step procedure to find $f'(x)$ for $f(x)=x^2+1$.

4) (15 pts) Find each derivative. You do not need to simplify your answer.

a) $\frac{d}{dx}\left(\frac{3}{x^2}\right)$

b) $\frac{d}{dx}\left(\frac{1}{\sqrt[4]{x^6}}\right)$

c) $\frac{d}{dx}\left(4x^2+2x-5\right)$

d) $\frac{d}{dx}\left[(x^2+1)(2x^2-3x+1)\right]$

e) $\frac{d}{dx}\left(\frac{\sqrt{x^5}}{3x^2-5x+1}\right)$

5) (15 pts) Find each derivative. You do not need to simplify your answer.

a) $\frac{d}{dx}\left(2e^{3x}\right)$

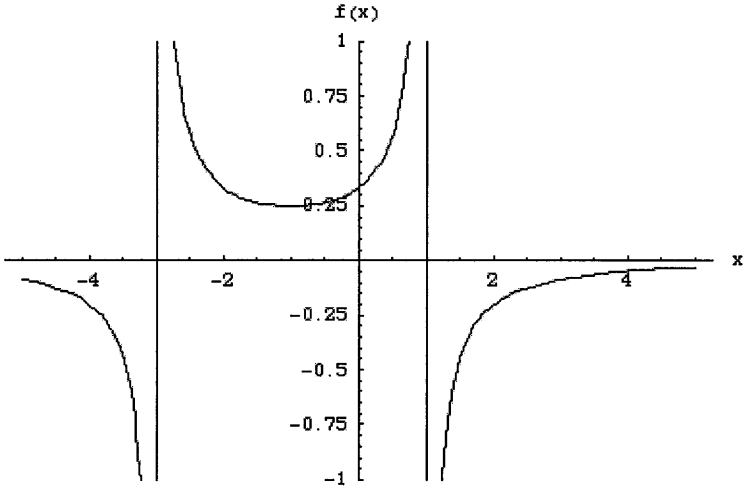
b) $\frac{d}{dx}\left[\ln(3x+5)\right]$

c) $\frac{d}{dx}\left(\frac{\ln x}{x^2}\right)$

d) $\frac{d}{dx}\left[\left(e^x+\ln x\right)^5\right]$

e) $\frac{d}{dx}\left(\left[\ln\left(x^3+7\right)\right]^2\right)$

6) (7 points) Use the graph of $f(x)$ to answer the following questions.



a) Find the intervals over which $f(x)$ is increasing.

b) Identify the intervals over which $f'(x) < 0$.

c) Identify the approximate x-coordinate of the point where $f'(x) = 0$.

d) Find the intervals over which the graph of $f(x)$ is concave upward.

e) Identify the intervals where $f''(x) < 0$.

f) Find any horizontal asymptote(s).

g) Find any vertical asymptote(s).

7) (3 pts) Find any inflection points of $g(x) = x^3 - 2x^2$.

8) (5pts) The price-demand function for a company is given by $p = 2x + 1$, and the cost function is given by $C(x) = x^2 - 2$. Find each of the following functions.

a) The revenue function, $R(x)$.

b) The profit function, $P(x)$.

c) The marginal profit function, $P'(x)$.

d) The average profit function, $\overline{P}(x)$.

e) The marginal average profit function, $\overline{P'}(x)$.

9) (4 pts) Find the equation of the line tangent to the graph of $f(x) = x^2\sqrt{3x+1}$ at $x = 1$.

10) (3 pts) Find the absolute extrema for $f(x) = x^4 + 4x$ on the interval [-2, 0].

11) (6 pts) Let $f(x) = \frac{x^3}{3} - x^2 - 3x$.

a) Find the critical value(s) of $f(x)$.

b) Find the intervals on which $f(x)$ is increasing, and decreasing.

c) Find any local extrema of $f(x)$.

12) (4 pts) Consider the equation $g(x) = \frac{x^2 - 6x + 9}{x^2 - 9}$.

a) Find the vertical asymptote(s). Your answer should be written as an equation of a line.

b) Find the horizontal asymptote(s). Your answer should be written as an equation of a line.

13) (3 pts) Find $\frac{dy}{dx}$ if $x^3 + 5x - 3y^2 + 5y = 0$.

14) (12 pts) Evaluate the indefinite integrals. You do not need to simplify your answer.

a) $\int (5x^3 + 1)\,dx$

b) $\int \frac{7}{x}\,dx$

c) $\int (4e^x + 3x)\,dx$

d) $\int \left(3\sqrt{x} - \frac{1}{\sqrt[3]{x^2}}\right)dx$

15) (9 pts) Evaluate the integrals using u-substitution. You do not need to simplify your answer.

a) $\int(3+x^2)2x\,dx$

b) $\int\frac{x^2}{x^3+5}dx$

c) $\int xe^{x^2+5}\,dx$

16) (6 pts) Evaluate the definite integrals. You do not need to simplify your answer.

a) $\int_0^3(x^2+2)dx$

b) $\int_0^1 e^{3x}\,dx$

17) (6 pts) Sales records from Darma Corp. indicate that the rate of change of the revenue (i.e. the marginal revenue) in dollars per unit is given by $R'(x) = 300 - 0.2x$, where x represents the number of units sold.

a) Find the total revenue function for the product. You can assume that $R(0) = 0$.

b) Find the revenue from the sale of 10 units.

Cheat Sheet

Derivative rules and definition:

1.) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$, provided the limit exists

2.) $\frac{d}{dx}\ C = 0$, for any constant C

3.) $\frac{d}{dx}\ x^n = nx^{n-1}$, for any real n

4.) $\frac{d}{dx}\ ku(x) = ku'(x)$, for any constant k

5.) $\frac{d}{dx}\ g(x) \pm h(x) = g'(x) \pm h'(x)$

6.) $\frac{d}{dx}\ g(x) \cdot h(x) = g(x) \cdot h'(x) + h(x) \cdot g'(x)$

7.) $\frac{d}{dx}\ \left[\frac{g(x)}{h(x)}\right] = \frac{h(x) \cdot g'(x) - g(x) \cdot h'(x)}{[h(x)]^2}$

8.) $\frac{d}{dx}\ [u(x)]^n = n[u(x)]^{n-1} \cdot u'(x)$, for any real n

9.) $\frac{d}{dx}\ e^{u(x)} = e^{u(x)}u'(x)$

10.) $\frac{d}{dx}\ ln[u(x)] = \frac{1}{u(x)}u'(x)$

Indefinite Integration rules:

1.) $\int kudu = k \int udu$, for any constant k

2.) $\int (f \pm g)dx = \int fdx \pm \int gdx$

3.) $\int u^n du = \frac{u^{n+1}}{n+1} + c$, for any real $n \neq -1$

4.) $\int e^u du = e^u + c$

5.) $\int \frac{1}{u} du = ln|u| + c$

Definite Integration rules:

1.) $\int_a^a f(x)dx = 0$

2.) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

3.) $\int_a^b kf(x)dx = k\int_a^b f(x)dx$, for k a constant

4.) $\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

5.) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

Fundamental Theorem of Calculus: If f is a continuous function on [a,b], and F is any antiderivative of f, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

Business Terms:

Total Cost $=C(x)=$ fixed costs + variable costs

Marginal Cost $= C'(x)$

Average Cost $= \overline{C}(x) = \dfrac{C(x)}{x}$

Marginal Average Cost $= \overline{C'}(x) = \dfrac{d}{dx}\overline{C}(x)$

Price Demand Function $= p(x)$

Total Revenue $= R(x) = x \cdot p(x)$,

Marginal Revenue $= R'(x)$

Average Revenue $= \overline{R}(x) = \dfrac{R(x)}{x}$

Marginal Average Revenue $= \overline{R'}(x) = \dfrac{d}{dx}\overline{R}(x)$

Total Profit $= P(x) = R(x) - C(x)$

Marginal Profit $= P'(x)$

Average Profit $= \overline{P}(x) = \dfrac{P(x)}{x}$

Marginal Average Profit $= \overline{P'}(x) = \dfrac{d}{dx}\overline{P}(x)$

Elasticity of Demand= For a given price p, the elasticity of p is $E(p) = \dfrac{-pf'(p)}{f(p)}$. If $E(p) > 1$, the demand is elastic, if $0 < E(p) < 1$, the demand is inelastic.