

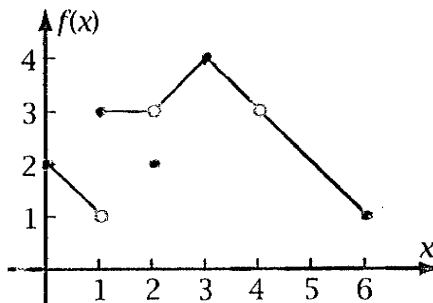
Part I: Each question is worth 2 points. Just record your final answer.

In problems 1-5, answer the questions using the graph of $f(x)$.

1. $\lim_{x \rightarrow 1^-} f(x) =$ _____

2. $\lim_{x \rightarrow 4} f(x) =$ _____

3. $f(2) =$ _____



4. For what values of x is f not continuous? _____

5. Is f differentiable at $x=3$? _____

6. $\lim_{x \rightarrow 3} \frac{x^2 + x - 6}{x + 3} =$ _____

7. An object's position at time t (in seconds) is given by the function $s(t) = 80t - 10t^2$ (in feet) for t between 0 and 8 seconds. Find the instantaneous velocity of the object when $t = 2$.

_____ feet / sec

In problems 8-10, evaluate the derivative. You do not have to simplify your answer.

8. Find $h'(x)$ when $h(x) = \frac{7x^5 - 3x^3 + 4x}{x^3}$ _____

9. $\frac{d}{dx}(e^x \ln x) =$ _____

10. Find y' if $y = \sqrt[3]{2x^3 - 3}$ _____

For problems 11 and 12 use the following function.

$$g(x) = \frac{5x^2 - 7}{x^2 + 2x - 15}$$

11. The equation for the horizontal asymptote is: _____

12. The equations for the vertical asymptotes are: _____ and _____

13. If \$6000 is invested 7% interest compounded continuously, how long would it take for the investment to double? (Round your answer to the nearest tenth)

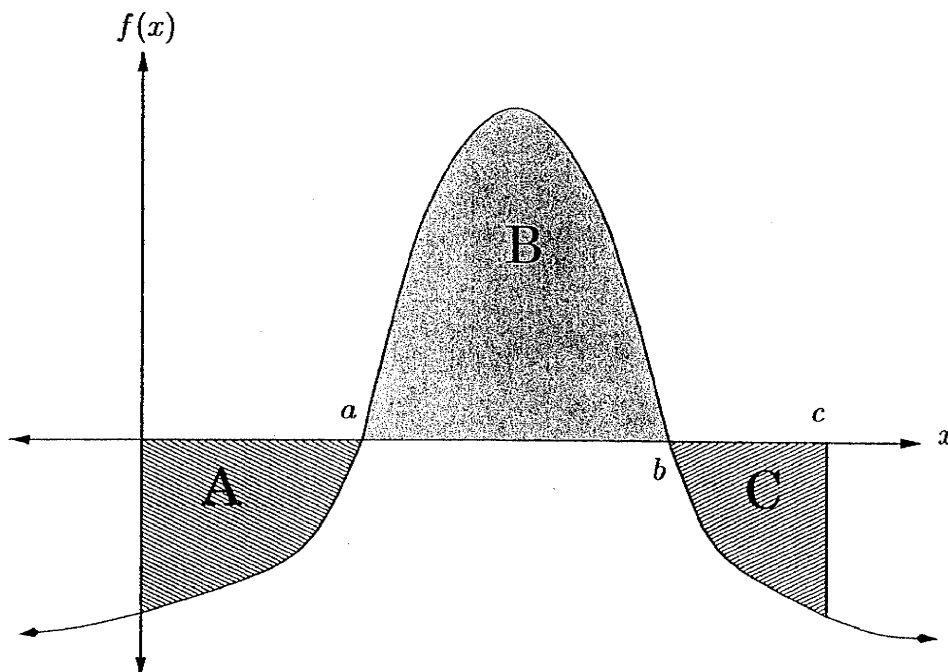
In problems 14, 15, and 16 evaluate the indefinite integrals.

14. $\int (8x - 7) dx =$ -----

15. $\int \left(e^x + \frac{1}{x} \right) dx =$ -----

16. $\int 2t(t^2 + 1) dt =$ -----

Problems 17 and 18 refer to the following figure with the indicated areas:



Area A = 4.5

Area B = 7

Area C = 3

17. $\int_0^c f(x) dx =$ -----

18. $\int_c^a f(x) dx =$ -----

Part II: 4 points each. For credit you must show all your work in a neat legible fashion.

For problems 19, 20, 21, use the following function

$$f(x) = x^3 - 9x^2 + 15x + 10$$

19. $f'(x) =$ _____

Find all the critical values.

The critical numbers are: _____

Using a sign chart and appropriate test numbers, find the increasing and decreasing intervals and local extrema of $f(x)$.

20. increasing intervals: _____ decreasing intervals: _____

The local maximum is _____ when $x =$ _____.

The local minimum is _____ when $x =$ _____.

21. $f''(x) =$ _____

Find the inflection point.

Find the intervals of concavity. Show your sign chart with appropriate test numbers.

The inflection point is (_____ , _____)

concave up: _____

concave down: _____

22. Use the given information to sketch the graph of f . Assume that f is continuous on its domain and that all intercepts are included in the table of values. Use dotted lines to represent the horizontal and vertical asymptotes.

Domain: All real x , except $x=1$;

$$f(0) = -2, \quad f(2) = 0;$$

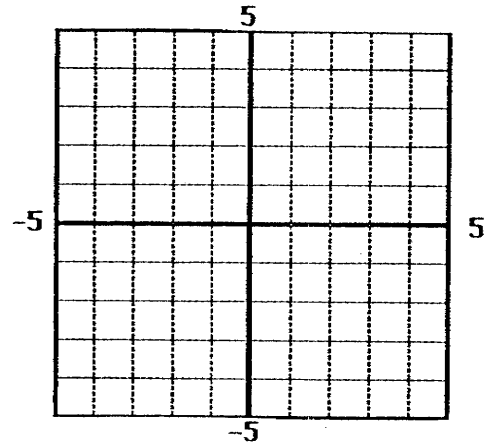
$f'(x) < 0$ on $(-\infty, 1)$ and $(1, \infty)$;

$f''(x) < 0$ on $(-\infty, 1)$;

$f''(x) > 0$ on $(1, \infty)$;

$$\lim_{x \rightarrow -\infty} f(x) = -1; \quad \lim_{x \rightarrow \infty} f(x) = -1$$

vertical asymptote: $x=1$



23. Given $h(x) = \frac{e^{2x}}{(x^2 - 3)^3}$ find $h'(x)$. Simplify your answer.

24. Find y' for $y=f(x)$ defined implicitly by the equation $y^2 - 5y + 4x^2 = 0$, and find the equation of the tangent line at the point $(1, 4)$.

25. Use u substitution to find $\int \frac{1}{3x-5} dx$.

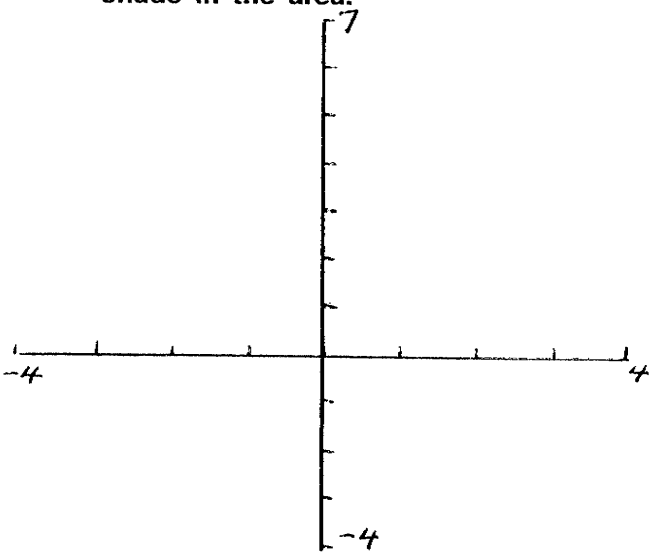
A) $u =$

B) $du =$

C) Final answer in terms of x .

$$\int \frac{1}{3x-5} dx = \underline{\hspace{4cm}}$$

26. Find the area between $y = x^2 - 3$ and $y = 2x$. Sketch a graph of the 2 functions and shade in the area.



Part III: Each problem is worth 8 points. For credit, you must show all your work in a neat legible fashion.

27. The price-demand equation and the cost function for the production of television sets are given respectively, by $x = 9000 - 30p$ and $C(x) = 150,000 + 30x$ where x is the number of sets that can be sold at a price of \$ p per set and $C(x)$ is the total cost (in dollars) of producing x sets

A) Express the price p as a function of x with domain $0 \leq x \leq 9000$. Use this function to find the revenue function and the marginal revenue function.

$p(x) =$ _____ $R(x) =$ _____ marginal revenue = _____

B) Find $R'(6,000)$ and interpret the results. $R'(6,000) =$ _____

C) Find the break-even points. You can do this algebraically or graphically.

The break-even points are:

Show algebra or sketch your graph.

(_____ , _____)

(_____ , _____)

D) Find the maximum revenue.

The maximum revenue is _____ when _____ TV's are produced.

28. A bicycle manufacturer has \$120 as unit cost (variable cost) and \$1,000 as a fixed cost. The price-demand equation is: $p = 300 - 0.2x$ for $0 \leq x \leq 1,500$

A) Find the revenue function: $R(x) =$ _____

Find the cost function: $C(x) =$ _____

Find the profit function: $P(x) =$ _____

B) What is the maximum profit, price to be charged to maximize profit and the production level that gives you the maximum profit? Use the 2nd derivative test to show that you have actually found the maximum.

The maximum profit is \$ _____ when _____

bicycles are produced at a price of \$ _____

29. The marginal average cost for producing x digital sports watches is given by

$$\bar{C}'(x) = -\frac{1200}{x^2} \quad \bar{C}(120) = 30$$

where $\bar{C}(x)$ is the average cost in dollars.

A) Set up an indefinite integral and find the average cost function.

$$\bar{C}(x) = \text{-----}$$

B) What is the total cost for 120 sports watches? _____

30. The total accumulated costs $C(t)$ and revenues $R(t)$ (in thousands of dollars), respectively, for a coal mine satisfy

$$C'(t) = 4 \quad \text{and} \quad R'(t) = 8e^{-0.1t}$$

where t is the number of years the mine has been in operation.

A) Find the useful life of the mine to the nearest year. Remember the value of t for which $C'(t) = R'(t)$ is called the useful life.

B) Set up a definite integral and find the total profit accumulated during the useful life of the mine.

CHAPTER 9

1. $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ Derivative of f at x, instantaneous rate of change and the slope of the tangent line
2. Point slope form for the equation of a line. $y - y_1 = m(x - x_1)$ or $y = m(x - x_1) + y_1$
3. Product Rule: If $y = f(x) = F(x)S(x)$ then $y' = FS' + SF'$
4. Quotient Rule: If $y = f(x) = \frac{T(x)}{B(x)}$ then $y' = \frac{BT' - TB' }{B^2}$
5. General Power Rule: If $y = [u(x)]^n$ then $y' = nu^{n-1}u'$
6. The word *marginal* refers to an instantaneous rate of change--that is, a derivative.
7. Revenue = x (#of items) times price Profit = revenue - cost
8. Break-even points: Where the Revenue and Cost are equal.

CHAPTER 10

1. Increasing and Decreasing Functions First Derivative Test (Local Extrema)

<u>f'(x)</u>	<u>f(x)</u>
+	Increases
-	Decreases

Let c be a critical value of f

a) if f'(c) changes from negative to positive at c, then f(c) is a local Min.

b) if f'(c) changes from positive to negative at c, then f(c) is a local Max.

2. Second-Derivative Test
Let c be a critical value for f(x)

<u>f'(c)</u>	<u>f''(c)</u>	<u>Graph of f is:</u>	<u>f(c)</u>
0	+	Concave up	Local minimum
0	-	Concave down	Local maximum
0	0	Test Fails	

3. Asymptotes

$y = b$ is a horizontal asymptote if $\lim_{x \rightarrow -\infty} f(x) = b$ or $\lim_{x \rightarrow \infty} f(x) = b$

Let $f(x) = \frac{n(x)}{d(x)}$, where both n and d are continuous at $x=c$. If, at $x=c$, the denominator $d(x)$ is 0 and the numerator $n(x)$ is not 0, then the line $x = c$ is a vertical asymptote for the graph of f.

4. The minimum average cost occurs when the average cost is equal to the marginal cost.

CHAPTER 11

1. **Continuous Compound Interest Formula.** $A = Pe^n$

2. **General Derivative Rules** a) $\frac{d}{dx}[f(x)]^n = n[f(x)]^{n-1} f'(x)$

b) $\frac{d}{dx} \ln[f(x)] = \frac{1}{f(x)} f'(x)$

c) $\frac{d}{dx} e^{f(x)} = e^{f(x)} f'(x)$

CHAPTER 12

1. **Indefinite Integral Formulas and Properties.** For k and C constants

1) $\int k \, dx = kx + C$

2) $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$

3) $\int kf(x) \, dx = k \int f(x) \, dx$

4) $\int [f(x) \pm g(x)] \, dx = \int f(x) \, dx \pm \int g(x) \, dx$

5) $\int e^x \, dx = e^x + C$

6) $\int e^{ax} \, dx = \frac{1}{a} e^{ax} + C$

7) $\int \frac{1}{x} \, dx = \ln|x| + C$

2. **Definite Integral Symbol for Functions with Negative Values**

If $f(x)$ is positive for some values of x on $[a, b]$ and negative for others, then

the definite integral symbol $\int_a^b f(x) \, dx$ represents the cumulative sum of the signed areas between the graphs of $y = f(x)$ and the x -axis where the areas above the x -axis are counted positively and the area below the x -axis are counted negatively.

3. **Fundamental Theorem of Calculus:** If f is a continuous function on the closed interval $[a, b]$ and F is any antiderivative of f , then

$$\int_a^b f(x) \, dx = F(x) \Big|_a^b = F(b) - F(a); \quad F'(x) = f(x)$$

CHAPTER 13

1. **Area between a Curve and the x Axis**

For $f(x) \geq 0$ over $[a, b]$: $Area = \int_a^b f(x) \, dx$

For $f(x) \leq 0$ over $[a, b]$: $Area = -\int_a^b f(x) \, dx$

2. **Area between two curves** $f(x) \geq g(x)$ over the interval $[a, b]$

$$Area = \int_a^b [f(x) - g(x)] \, dx$$