Math 1070 Uniform Final Exam

May 6, 2006

Print your name: _____

Circle your section below.

Lana	Vis	Pilliteri
001	002	003
MW 8:30 AM	T/R 10:00 AM	M/W 11:30 AM

Instructions:

- Put your name on this page and on the next page.
- Circle your section number above.
- The exam is closed notes and closed book. A sheet of formulas is provided at the end of the exam. You also may use your calculator during the exam.
- Answer each question in the space provided. Show your work when possible. Partial credit is possible on some questions, but only if you show your work.
- You may not communicate with anyone other than the instructor during the exam. The instructor may help to clarify the meaning of a question but is not allowed to give you hints, verify formulas, or tell you if you have done the problem correctly.
- Be neat. If the grader cannot understand what you have written, you will not receive credit.

DO NOT WRITE BELOW THIS LINE

Page 1 (10 pts) _____

Page 2 (10 pts) _____

Page 3 (10 pts) _____

Page 4 (10 pts) _____

Page 5 (18 pts) _____

Page 6 (9 pts) _____

Page 7 (8 pts) _____

Page 8 (9 pts) _____

Page 9 (16 pts) _____

Total (100 pts) _____

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Part 1 - Short Answer Questions (2 points each)

Functions

1. Given $f(x) = \sqrt{x-3}$ and $g(x) = x^2 + 1$, find $\frac{g(-5)}{f(12)}$. Simplify your answer.

2. Find the minimum value of the function $f(x) = x^2 + 2x + 5$.

3. If $e^{5x} = 2$, then the **exact** value of x is _____.
(don't use your calculator)

4. In slope-intercept form ($y = mx + b$), write the equation of the line that passes through the point (2, -3) and has a slope of $\frac{3}{2}$.

5. Solve the following equation for x:

$$\log_{10}(x+2) - \log_{10}(x-4) = 1$$

Finance

6. A loan of $300 was repaid at the end of 4 months with a check for $312. What was the annual simple interest rate on this loan?

7. How long will it take for an investment to double if it is invested at 6.3% compounded continuously?

8. If an account at 5.2% compounded daily grows to $2500 after 3 years, what was the amount of the initial deposit?

9. A business has a $100,000, 20-year mortgage at 7.5% compounded annually. What is the annual payment?

10. If a loan at 8.3% compounded monthly is to be paid off in 24 monthly payments of $150, how much can be borrowed?

Systems of Linear Equations and Matrices

11. If $A=\begin{bmatrix}2 & 3\\ 1 & 4\end{bmatrix}$, find A^{-1}.

12. If $\begin{bmatrix}2 & 3\\ 1 & 4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}-2\\ 5\end{bmatrix}$, find x_1 and x_2. Hint: consider using your answer to the previous problem.

13. Solve the following system by any method.
$$\begin{aligned}2x+3y&=12\\ x-\ y&=3\end{aligned}$$

14. Find the reduced-row echelon form of the augmented matrix $\left[\begin{array}{ccc|c}2 & 3 & 1 & -2\\ 0 & 1 & 4 & 2\\ 0 & -2 & -8 & -4\end{array}\right]$.

15. Write the solution to the linear system represented by the matrix in the previous problem.

Counting and Probability

16. A random question. In how many ways can the letters R, A, N, D, O, M be written?

17. An automatic teller machine requires that each customer enter a four-digit personal identification number (PIN) when he or she inserts a bank card. If any digits (from 0 to 9) can be used and repetition is allowed, how many different PIN's are possible?

18. A company wishes to use the services of 3 different banks in a city. If 12 banks are available, in how many ways can it choose the 3 banks?

19. In the U.S. Senate, there are currently 14 women and 86 men. In how many ways can a committee consisting of 2 women and 4 men be selected from the members of the senate if each person on the committee will have equal standing?

20. If a pair of six sided dice are rolled, what is the probability of rolling a 10?

Part 2 - Application Problems and Multipart Problems

1. (8 pts) Consider the function $f(x) = \dfrac{3x}{(x-3)(x+2)}$

   A. What is the domain of f(x)?

   B. Evaluate f (-2).

   C. Find any horizontal asymptote(s) of f(x).

   D. Find any vertical asymptote(s) of f(x).

2. (10 pts) Consider the price-demand function $p(x) = 250 - 20x$, where $p(x)$ is the wholesale price at which x thousand stereo systems can be sold. Assume that $0 \leq x \leq 10$.

   A. Find the wholesale price when ten thousand stereo systems are sold.

   B. Find the revenue function.

   C. Find the revenue generated by selling five thousand stereo systems.

   D. Suppose the cost of producing x thousand stereo systems is given by the cost function C(x) =150+4x. Find the profit function.

   E. Graph the profit function over the interval $0 \leq x \leq 10$, with appropriately scaled axes. Neatness counts.

3. (9 pts) Larry, Moe, and Curly decide to save for retirement.

   A. If Larry makes annual deposits of $2000 for 5 years into an account earning 12.1% compounded annually, and then allows the money to earn interest for 40 years, how much will he have at the end of the forty-five years?

   B. If Moe wishes to make annual withdrawals of $60,000 for twenty-five years from an ordinary annuity earning 7.6% compounded annually, how much should he plan to have in the annuity at the start of those twenty-five years?

   C. If Curly wants to make annual deposits into an ordinary annuity earning 8.4% interest, compounded annually for forty years and have a balance of $2,000,000 after the forty years, what annual deposit should he make?

4. (8 points) A company manufactures two models of coffee tables. The first table has labor costs of $30.00 and material costs of $40.00, while the second table has labor costs of $20.00 and material costs of $30.00.

   A. During a particular week, $1200 of funding is allocated to labor and $1800 is allocated to materials. What quantities of each model of coffee table should be manufactured to completely use the allocated funding?

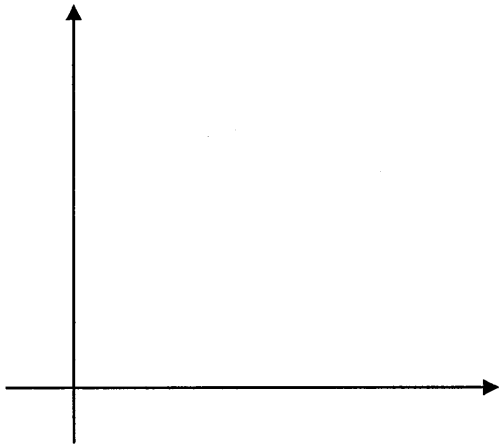
   B. During a different week, $1280 is allocated to labor and $1720 is allocated to materials. What quantities of each model of coffee table should be manufactured to completely use the allocated funding?

5. (9 points) Solve the following linear programming problem geometrically.

Maximize $f = 3x + 2y$ subject to the constraints:

$$x + y \leq 4$$
$$2x + y \leq 6$$
$$x \geq 0$$
$$y \geq 0$$

a. Carefully graph the constraints of this problem labeling the corner points and shading in the feasible region.



b. Evaluate the objective function at each corner point of the feasible region. You can use your calculator to help you evaluate the objective function at the corner points.

Corner Points	Objective Function

c. The maximum value of f is _____.

The maximum occurs when $x =$ _____ and $y =$ _____.

6. (8 points) **SET UP THIS PROBLEM. DO NOT SOLVE IT.**
A company makes two chemicals: Chemical A and Chemical B. One pound of Chemical A takes 62 hours to mix and 78 hours to package; one pound of Chemical B takes 42 hours to mix and 48 hours to package. The mixing department has at most 6920 hours available each year and the packaging department has at most 8280 hours available each year. Due to storage problems, a maximum of 100 pounds of chemical A can be produced each year, and a maximum of 150 pounds of chemical B can be produced each year. The profit for one pound of Chemical A is $76 and the profit for one pound of Chemical B is $48. Set up, but DO NOT SOLVE, the linear programming problem that you would use to determine how many pounds of each chemical should be produced in order to maximize the companies profit.

Let x = _____

Let y = _____

The objective function is _____

The constraints are: _____

_____

_____

_____

_____

7. (8 points) From a survey involving 500 people, a market research company found that 235 people owned a dog, 210 people owned a cat, and 95 owned both a dog and a cat. If one of these 500 people is selected at random, what is the probability that

A. The person owns a dog or a cat.

B. The person owns neither a dog nor a cat.

C. The person does not own a dog.

D. If you are told that the person owns a dog, what is the probability that they also own a cat.

Facts and Formulas

Algebra

$$a^x \cdot a^y = a^{x+y} \quad \frac{a^x}{a^y} = a^{x-y} \quad (a^x)^y = a^{xy} \quad (\frac{a}{b})^x = \frac{a^x}{b^x} \quad (ab)^x = a^x b^x$$

$$a^x = a^y \text{ iff } x = y \quad a^x = b^x \text{ iff } a = b, x \neq 0$$

$$\log_b 1 = 0 \quad \log_b b = 1 \quad \log_b b^x = x \quad b^{\log_b x} = x \ \ (x > 0)$$

$$\log_b(xy) = \log_b x + \log_b y \quad \log_b(x^n) = n \log_b x \quad \log_b(\frac{x}{y}) = \log_b x - \log_b y \quad \log_b x = \log_b y \text{ iff } x = y$$

Cost function: $C = \text{(fixed costs)} + \text{(variable costs)} = a + bx$

Price-demand: $p = m - nx$

Revenue function: $R = xp = x(m - nx)$

Profit function: $P = R - C = x(m - nx) - (a + bx)$

Finance

Simple interest: $I = Prt$

Simple interest: $A = P(1 + rt)$

Compound interest: $A = P\,(1 + i)^n$ or $A = P\left(1 + \frac{r}{m}\right)^{mt}$

Continuous interest: $A = Pe^{rt}$

Annual percentage yield: $APY = \left(1 + \frac{r}{m}\right)^m - 1$

Future value ordinary annuity: $FV = PMT\frac{(1+i)^n - 1}{i}$

Present value ordinary annuity: $PV = PMT\frac{1-(1+i)^{-n}}{i}$

Counting and Probability

$$\text{Permutations: } {}_nP_k = \frac{n!}{(n-k)!} \qquad \text{Combinations: } {}_nC_k = \frac{n!}{(n-k)!\ \ k!}$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$