

**FILE COPY**

# MATH 1070 UNIFORM FINAL EXAM

May 10, 2003

Print your name \_\_\_\_\_

Circle your section:

Butler	Doom	Gryboski	Sullivan	Witulski	Yoo
Online	MW 10:00	MW 11:30	MW 4:00	TTh 8:30	TTh 2:30

**Instructions:** Read the problems carefully. If you do not understand a question, ask for clarification. You have three hours to complete the exam. You may use a calculator and the page of formulas provided with the exam. The maximum score for this exam is 100 points: 50 points for Part 1 and 50 points for Part 2. To receive credit for Part 1 questions, you must transfer your answers to the answer sheet. To receive credit for Part 2 questions, you must show all work. Good luck!

DO NOT WRITE BELOW THIS LINE

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Part 1: Short Answer (50 pts) \_\_\_\_\_

26. (10 pts) \_\_\_\_\_

27. (10 pts) \_\_\_\_\_

28. (10 pts) \_\_\_\_\_

29. (10 pts) \_\_\_\_\_

30. (10 pts) \_\_\_\_\_

Total (100 pts): \_\_\_\_\_

# ANSWER SHEET FOR PART 1

You must transfer your answers to questions in Part 1 to this sheet.

Print your name and section \_\_\_\_\_

- |           |           |
|-----------|-----------|
| 1. _____  | 14. _____ |
| 2. _____  | 15. _____ |
| 3. _____  | 16. _____ |
| 4. _____  | 17. _____ |
| 5. _____  | 18. _____ |
| 6. _____  | 19. _____ |
| 7. _____  | 20. _____ |
| 8. _____  | 21. _____ |
| 9. _____  | 22. _____ |
| 10. _____ | 23. _____ |
| 11. _____ | 24. _____ |
| 12. _____ | 25. _____ |
| 13. _____ |           |

Total points \_\_\_\_\_

Print your name and section \_\_\_\_\_

SHORT ANSWER (2 POINTS EACH, NO PARTIAL CREDIT, TRANSFER YOUR ANSWERS TO THE ANSWER SHEET)

Functions

1. Find the equation of the line, in point-intercept form, that passes through the point  $(-4, 6)$  with a slope of 2.
2. Solve the equation  $2^{2x+1} - 2^{3x-4} = 0$  for  $x$ .
3. What value(s) of  $x$  satisfy the equation  $\log_8(x^2 - 1) = 1$ ?
4. What is the domain of the function  $g(x) = \frac{5}{x^2 + 4x - 32}$ ?
5. Find the minimum value of the function  $f(x) = x^2 - 2x + 5$ .

Probability

6. How many six-letter code words can be made from the first ten letters of the alphabet (A, B, C, D, E, F, G, H, I, J)? Assume that letters may *not* be repeated.

7. A lock on a suitcase has four wheels, each of which can display one of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. What is the probability of randomly guessing the correct combination for the lock? Assume that numbers may be repeated.

8. What is the probability that the parents of a four-child family have one boy and three girls? Assume multiple births do not occur and boys and girls are equally likely.

The following table, to be used in problems 9 and 10, shows the number of women/men and Republicans/Democrats that attended a conference.

	Women	Men
Democrats	20	15
Republicans	10	30

9. What is the probability that you randomly meet a woman Democrat at the conference?

10. What is the probability that you randomly meet a man or a Republican at the conference?

#### Finance

11. If you borrow \$2000 at 10.5% simple interest, how much do you owe (principal and interest) at the end of three years?

12. What is the present value of an ordinary annuity that pays \$500 per month for ten years? Assume the annuity earns 5.5% annual interest compounded monthly.

13. If you make a single payment to an account that earns 13% compounded annually, how long will it take your money to triple in value?

14. How much would you have to invest monthly at 7% compounded monthly if you plan to have \$10,000 in five years?

15. How much will \$1000 be worth at the end of 40 years if it is invested at 8% compounded annually?

Linear Systems and Inequalities

16. Solve the following linear system using any method.

$$\begin{aligned} 2x + y &= -3 \\ -4x + 2y &= 4 \end{aligned}$$

17. If  $C$  is a  $4 \times 3$  matrix and  $D$  is a  $3 \times 7$  matrix, what are the dimensions of the product of  $CD$ ?

18. Perform the row operation  $2R_1 + R_2 \rightarrow R_2$  on the matrix  $\left( \begin{array}{cc|c} 2 & 0 & 5 \\ -3 & 1 & -4 \end{array} \right)$ .

19. Find  $BA$  given that

$$A = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad B = (-2 \ 1 \ 3).$$

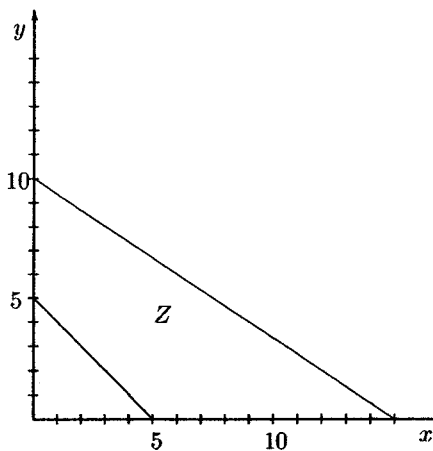
20. Write out the equations of the linear system described by the matrix  $\left( \begin{array}{cc|c} 2 & -1 & 3 \\ 1 & 3 & -2 \end{array} \right)$ .

21. The corner points of a bounded feasible region are  $O = (0, 0)$ ,  $A = (0, 14)$ ,  $B = (2, 8)$ , and  $C = (4, 0)$ . Find the maximum value of the objective function  $P = 3x + 2y$  over the region OABC.

Use the figure to the right, showing the region  $Z$ , for problems 22 and 23.

22. Find the maximum value of the objective function  $P = 2x - 3y$  over the region  $Z$ .

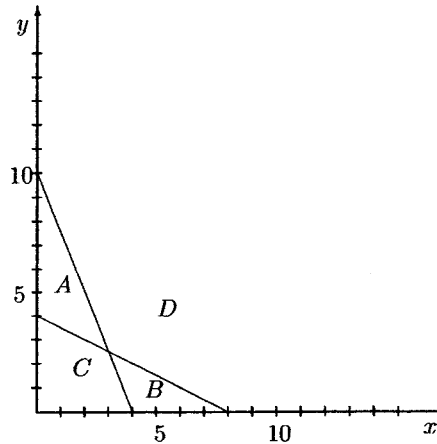
23. Find the minimum value of the objective function  $P = -x + y$  over the region  $Z$ .



Use the figure to the right for problems 24 and 25.  
 A feasible region is described by the inequalities

$$\begin{aligned} 5x + 10y &\leq 40 \\ 10x + 4y &\leq 40, \\ x &\geq 0 \\ y &\geq 0. \end{aligned}$$

24. Which of regions  $A$ ,  $B$ ,  $C$ , or  $D$  is described by the system of inequalities?  
 25. Find the corner points of the feasible region.



APPLICATIONS (10 POINTS EACH, PARTIAL CREDIT)

26. Consider the price-demand function  $p(x) = 250 - 20x$ , where  $p(x)$  is the wholesale price at which  $x$  thousand stereo systems can be sold. Assume that  $0 \leq x \leq 10$ .

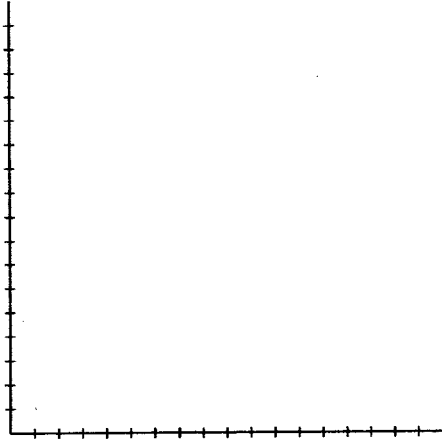
(a) Find the revenue function.

(b) Find the wholesale price for ten thousand stereo systems.

(c) Find the revenue generated by selling five thousand stereo systems.

(d) Suppose the cost of producing  $x$  thousand stereo systems is given by the cost function  $C(x) = 150 + 4x$ . Find the profit function.

(e) Graph the profit function with appropriately scaled axes.



27. At the end of each year for 30 years, you put \$3000 in an account that earns an average of 14.5% compounded annually.

(a) How much will you have in your account at the end of 30 years?

(b) At the end of 30 years, assume you withdraw \$1,000,000 from the account and put it in an annuity that pays 5% compounded monthly. If you receive monthly payments from this new annuity for the next 30 years, how much will be in each payment if there is a zero balance after 30 years?

(c) How much total interest will you have earned in the annuity of part (b)?

28. Three families order pizza for dinner. The first family orders one pizza, a large bottle of Coke, and pays \$14. The second family orders two pizzas, one order of breadsticks, two large bottles of Coke, and pays \$36. The third family orders one pizza, two orders of breadsticks, and pays \$24.

(a) Let  $x_1, x_2, x_3$  represent the cost of a pizza, an order of breadsticks, and a large bottle of Coke, respectively. Write a system of linear equations that describes the orders of each family.

(b) Solve the linear system in part (a). What is the cost of a pizza, an order of breadsticks, and a large bottle of Coke?

29. The officers of a high school senior class are planning to rent buses and vans for a class trip. Each bus can transport 40 students, requires 3 chaperones, and costs \$1200. Each van can transport 8 students, requires 1 chaperone, and costs \$100. The officers must devise a plan to transport at least 400 students with at most 36 chaperones that will minimize transportation costs.

(a) Define the variables of the problem.

(b) Find the objective function and constraints for the problem.

(c) Find the corner points of the feasible region.

(d) How many vans and how many buses must be used to minimize the cost of the trip?

(e) What is the minimum cost of the trip?

30. A survey of 150 city residents reveals that 70 people read the daily morning newspaper, 90 people read the daily evening newspaper, and 40 people read both newspapers.

(a) Draw a Venn diagram that describes the readership of the residents.

(b) What is the probability that a randomly selected person reads a daily newspaper?

(c) What is the probability that a randomly selected person does not read a daily newspaper?

(d) What is the probability that a randomly selected person reads exactly one daily newspaper?

## Facts and Formulas

### Algebra

$$a^x \cdot a^y = a^{x+y} \quad \frac{a^x}{a^y} = a^{x-y} \quad (a^x)^y = a^{xy} \quad \left(\frac{a}{b}\right)^x = \frac{a^x}{b^x} \quad (ab)^x = a^x b^x$$

$$a^x = a^y \text{ iff } x = y \quad a^x = b^x \text{ iff } a = b, x \neq 0$$

$$\log_b 1 = 0 \quad \log_b b = 1 \quad \log_b b^x = x \quad b^{\log_b x} = x \quad (x > 0)$$

$$\log_b(xy) = \log_b x + \log_b y \quad \log_b(x^n) = n \log_b x \quad \log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y \quad \log_b x = \log_b y \text{ iff } x = y$$

$$\text{Cost function: } C = (\text{fixed costs}) + (\text{variable costs}) = a + bx$$

$$\text{Price-demand: } p = m - nx$$

$$\text{Revenue function: } R = xp = x(m - nx)$$

$$\text{Profit function: } P = R - C = x(m - nx) - (a + bx)$$

### Finance

$$\text{Simple interest: } I = Prt$$

$$\text{Simple interest: } A = P(1 + rt)$$

$$\text{Compound interest: } A = P\left(1 + \frac{r}{m}\right)^n \text{ or } A = P\left(1 + \frac{r}{m}\right)^{mt}$$

$$\text{Continuous interest: } A = Pe^{rt}$$

$$\text{Annual percentage yield: } APY = \left(1 + \frac{r}{m}\right)^m - 1$$

$$\text{Future value ordinary annuity: } FV = PMT \frac{(1+i)^n - 1}{i}$$

$$\text{Present value ordinary annuity: } PV = PMT \frac{1 - (1+i)^{-n}}{i}$$

### Counting and Probability

$$\text{Permutations: } {}_n P_k = \frac{n!}{(n-k)!}$$

$$\text{Combinations: } {}_n C_k = \frac{n!}{(n-k)! k!}$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$