

Name _____

Basic Algebra

1. (2 points each) Simplify each expression.

a. $2(3x + 2)^2 - (3x - 6)$

b. $\frac{2x}{x^2 - 16} - \frac{3}{x^2 + 8x + 16}$

2. (2 points each) Simplify. Write your answer with all positive exponents.

a. $\frac{-18a^{-3}b^8c}{24a^{-5}b^{10}c}$

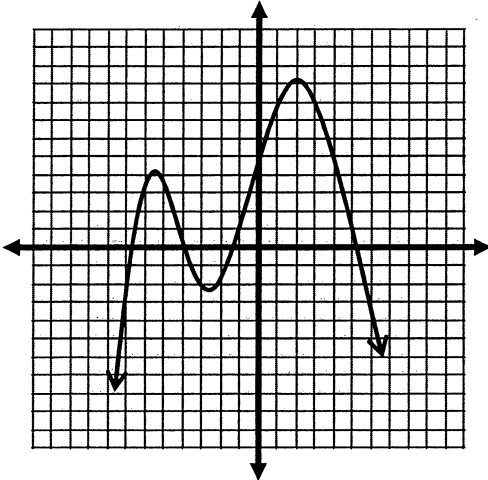
b. $(3a^{-3}b^5)^{-4}$

3. (2 points) Factor Completely: $18x^3 - 15x^2 - 12x$

Elementary Functions

4. (2 points each) Indicate whether each graph represents the graph of a function. Circle the appropriate answer below. Briefly explain, in one sentence or less, why you know that your answer is correct.

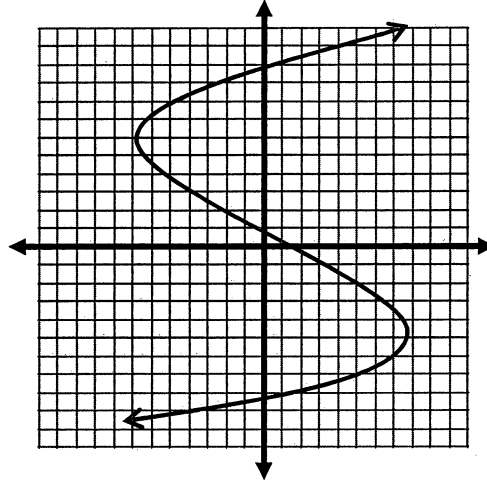
a.



Circle One: **Function** **Not a Function**

Explain:

b.



Circle One: **Function** **Not a Function**

Explain:

5. (2 points each) Find the domain of each function. Write your answer using interval notation.

a. $f(x) = \frac{x + 1}{x^2 - 11x + 24}$

b. $g(x) = \frac{1}{\sqrt{2x - 5}}$

6. (2 points each) Given the price demand function $p(x) = 75 - 3x$; for $1 \leq x \leq 20$.

a. Write the revenue function $R(x)$. Simplify.

b. If the cost is given by $C(x) = 125 + 16x$, write the Profit function $P(x)$. Simplify.

7. (2 points) Find the slope and y-intercept of $2x - 3y = 9$.

8. (2 points) Find the x and y intercepts of $-3x + 2y = 12$.

9. At a price of \$5.00, the supply of soybeans is 5 bushels, while the demand is 6 bushels. When the price increases to \$8.00, the supply increases to 4 bushels while the demand decreases to 5 bushels.

a. (2 points) Find the price-supply equation.

b. (2 points) Find the price-demand equation.

c. (3 points) Find the equilibrium point. Write your answer as an ordered pair of numbers.

10. Given the function $f(x) = x^2 - 6x + 5$

- a. (2 points) Find the vertex of the function.
- b. (2 points) Find the x-intercepts
- c. (2 points) Find the y-intercept
- d. (2 points) Find the interval(s) where $f(x)$ is increasing
- e. (2 points) Find the interval(s) where $f(x)$ is decreasing
- f. (4 points) Sketch a graph of $y = f(x)$. Label at least 4 points including any intercepts.

Additional Elementary Functions

11. (2 points each) Given $f(x) = x^4 - 4x^3 + 3x^2$.

- a. What is the degree of the function?
- b. Find the y-intercept algebraically.
- c. Find any x-intercepts algebraically.

12. Given $g(x) = \frac{2x+4}{x-3}$.

- a. (2 points) What is the domain of the function?
- b. (2 points) Find the y-intercept algebraically.
- c. (2 points) Find any x-intercepts algebraically.
- d. (2 points) Find any vertical asymptotes.
- e. (2 points) Find any horizontal asymptotes.
- f. (4 points) Sketch a graph of the function $y = g(x)$. Sketch and label the asymptotes and label all points of intersection.

13. (2 points each) Rewrite in exponential form

a. $\log_2 32 = 5$

b. $\ln 1 = 0$

14. (2 points each) Rewrite in logarithmic form.

a. $9^{\frac{1}{2}} = 3$

b. $5^0 = 1$

15. (2 points each) Evaluate the following logarithmic expressions.

a. $\log_3 9$

b. $\ln e^3$

16. (2 points each) Solve for x .

a. $\log_b 2x + \log_b x = 3\log_b 2$

b. $\log_3\left(\frac{1}{3}\right) = x$

Sequences and Series

17. (2 points) Let a_1, a_2, a_3, \dots be an arithmetic sequence. Given $a_1 = -1$ and $a_{10} = 17$ find a_{50} (the 50th term of the sequence).

18. (2 points) Write the first four terms of the sequence with general term $a_n = 2^{n-2}$.

19. (2 points) Find the general term of a sequence whose first four terms are given.

a. 3, 5, 7, 9, ...

b. 1, 3, 9, 27, ...

20. (2 points each) Write the following series using summation notation (sigma notation) with the summing index k starting at $k = 1$.

a. $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$

b. $3 + 6 + 12 + 24$

21. (2 points) Evaluate $S_4 = \sum_{k=1}^4 (3)^{k-1}$

Mathematics of Finance

In the next four problems you need to solve for the variable and leave your answer as you would put it into your calculator.

22. (4 points) A check for \$40,000 was used to retire a 6 year \$32,000 loan. What simple annual rate of interest was charged? Simplify and reduce your answer (represent it as a fraction.)

23. (4 points) How long will it take for \$20,000 to grow to \$100,000 if it is invested at 7% compounded monthly?

24. (4 points) How much do you need to invest now to have \$100,000 in 18 years if it is invested in an account that receives 5% compounded continuously?

25. (4 points) A man wishes to save for a car. He decides that he will invest semiannually into an account that receives 7% compounded semiannually. How much should he deposit semiannually to have \$40,000 in 6 years for his car?

Systems of Linear Equations and Matrices

26. (4 points) Solve the following system of equations by your choice of either the substitution method or the elimination method. Show all work.

$$\begin{cases} 2x + 2y = 7 \\ 2x + 4y = 8 \end{cases}$$

27. (4 points) Solve the system of equations by Gauss-Jordan elimination using augmented matrices. Show all steps and identify the row operations that you use.

$$\begin{cases} 3x - 3y = 3 \\ 2x - 4y = -6 \end{cases}$$

28. (2 points each) Given $A = \begin{bmatrix} -3 & 2 \\ 2 & -2 \\ -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 1 \\ 3 & -1 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$.

a. If possible find $3B + C$. Show your work. If this is not possible explain why not.

b. If possible find AB . Show your work. If this is not possible explain why not.

29. (4 points) A street vendor sells bags of popcorn and pretzels. A bag of popcorn sells for \$2.00 and pretzels sell for \$1.50 each. On a particular afternoon the vendor sold 52 items and had revenue of \$93.00. How many bags of popcorn and how many pretzels did the vendor sell? Set up the linear system that represents this problem and solve it by your choice of methods. Show your work.

Sets and Counting

30. (2 pts. each) Given $A = \{1, 3, 5\}$, $B = \{1, 2, 3\}$, $C = \{2, 4, 6, 8\}$, and $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ find each of the following.

a. $A \cup B =$	b. $A \cap C =$
c. $C' =$	d. $(A \cap B)' =$

31. (2 points) A particular new car is available with 4 choices of color, 2 choices of transmission, 3 types of interior and 2 types of engine. How many different variations of this model car are possible? Show work and write your answer as a whole number.

Formula Sheet

Simple Interest Formulas:

$$I = Prt$$

$$A = P + Prt$$

Compound Interest Formula:

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

Continuous Compound Interest Formula

$$A = Pe^{rt}$$

Future Value of an Ordinary Annuity:

$$FV = PMT \frac{\left(1 + \frac{r}{m}\right)^{mt} - 1}{\frac{r}{m}}$$

Properties of Logarithmic Functions:

$$\log_b x = y \text{ if and only if } b^y = x$$

$$\log_b(MN) = \log_b(M) + \log_b(N) \quad \log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N \quad \log_b M^p = p \log_b M$$

Arithmetic Sequence and Series Formulas:

$$\text{The } n\text{th term of an Arithmetic Sequence: } a_n = a_1 + (n-1)d$$

$$\text{The sum of a Finite Arithmetic Series: } S_n = \frac{n}{2}[2a_1 + (n-1)d] \text{ or } S_n = \frac{n}{2}(a_1 + a_n)$$

Geometric Sequence and Series Formulas:

$$\text{The } n\text{th term of a Geometric Sequence: } a_n = a_1 r^{n-1}$$

$$\text{The sum of a Finite Geometric Series: } S_n = \frac{a_1(r^n - 1)}{r - 1} \text{ or } S_n = \frac{ra_n - a_1}{r - 1}$$

Combinations and Permutations:

$$\text{Combinations: } {}^n C_r = \frac{n!}{r!(n-r)!}$$

$$\text{Permutations: } {}^n P_r = \frac{n!}{(n-r)!}$$