

**FILE COPY**

# MATH 1070 UNIFORM FINAL EXAM

December 6, 2003

Print your name \_\_\_\_\_

Circle your section:

Butler	Creek	Hebert	Jabri	Sullivan	Sullivan	Witulski
Online	TTh 5:30	TTh 8:30	TTh 10:00	MW 11:30	MW 1:00	MW 10:00

**Instructions:** Read the problems carefully. If you do not understand a question, ask for clarification. You have three hours to complete the exam. You may use the page of notes and formulas attached to the exam and a calculator. The maximum score for this exam is 100 points: 50 points for Part 1 and 50 points for Part 2. To receive credit for Part 1 questions, you must transfer your answers to the answer sheet. To receive **partial** credit for Part 2 questions, you must **show all work**. Good luck!

DO NOT WRITE BELOW THIS LINE

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Part 1: Short Answer (50 pts) \_\_\_\_\_

26. (10 pts) \_\_\_\_\_

27. (10 pts) \_\_\_\_\_

28. (10 pts) \_\_\_\_\_

29. (10 pts) \_\_\_\_\_

30. (10 pts) \_\_\_\_\_

Total (100 pts): \_\_\_\_\_

# ANSWER SHEET FOR PART 1

You must transfer your answers to questions in Part 1 to this sheet.

Print your name and section \_\_\_\_\_

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25. \_\_\_\_\_

Total points \_\_\_\_\_

Print your name and section \_\_\_\_\_

SHORT ANSWER (2 POINTS EACH)

Functions

1. In slope-intercept form ( $y = mx + b$ ), write the equation of the line that passes through the point  $(-2, 8)$  and has a slope of  $4/5$ .
2. What is the domain of the function  $f(x) = \sqrt{7 - 2x}$ ?
3. Find the vertex of the graph of  $f(x) = -(x - 4)^2 + 6$  and state whether it corresponds to a minimum or maximum.
4. Given that  $f(x) = 3x^2 + 2x + 1$  and  $g(x) = 6x + 5$ , evaluate  $f(3)/g(-6)$ .
5. Solve the following equation for  $x$ :  $\log_{10}(x + 2) - \log_{10}(x - 4) = 1$ .

Probability

6. How many six-letter code words are possible using the first ten letters of the alphabet, if letters can be repeated?
7. A combination lock has 4 wheels, each labelled with the digits 0 through 9. How many combinations are possible if no number is repeated?
8. From a standard 52-card deck, how many 3-card hands contain all diamonds?
9. A committee of 5 people is chosen randomly from a group of 20 women and 15 men. In how many ways can a committee consisting of two men and 3 women be chosen?
10. Suppose you toss three fair coins, what is the probability of obtaining exactly two heads?

Finance

11. What is the value of an annuity at the end of 20 years if \$2000 is deposited each year into an account earning 8.5% compounded annually?

12. How much should you invest now at 10% compounded quarterly to have \$8000 toward the purchase of a car in 5 years ?
13. Two parents set up an annuity in order to have \$140,000 in 17 years for their child's college education. How much should be paid semi-annually into an account paying 6.8% semi-annually in order to reach this goal?
14. Suppose you borrow \$100 for three months. How much (simple) interest should be charged so that at the end of three months, you owe \$105?
15. What is the annual percentage yield in a money market account that pays 6.35% compounded monthly?

Linear Systems

16. Find the point of intersection of the lines  $x + 3y = 1$  and  $3x - 2y = 14$ .
17. True or False: The following matrix is in reduced (row echelon) form:

$$\begin{pmatrix} 1 & 0 & -2 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

18. Evaluate the product

$$\begin{pmatrix} 3 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

19. Evaluate  $C + BA$ , where

$$A = \begin{pmatrix} 1 & 1 & -3 \\ 4 & 5 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 2 \\ 0 & 4 \\ 1 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 7 & 3 & -1 \\ 2 & 11 & -2 \\ 1 & 0 & 4 \end{pmatrix}.$$

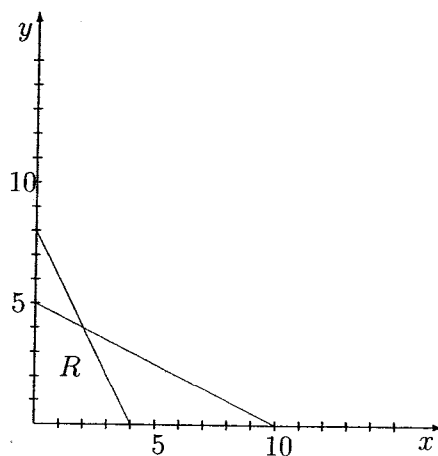
20. Find the inverse of the matrix

$$\begin{pmatrix} 1 & -1 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}.$$

Linear Programming

Refer to the figure below for problems 21–23.

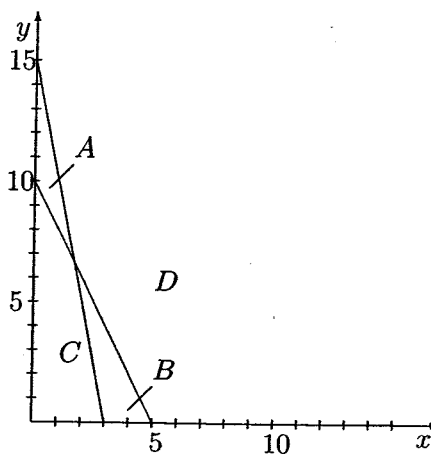
$$\begin{aligned} 6x + 3y &\leq 24 \\ 3x + 6y &\leq 30 \\ x, y &\geq 0 \end{aligned}$$



21. Let  $R$  be the region shown in the figure. Is  $R$  a bounded region?
22. Given  $P = 3x - 2y$ , find the maximum value of  $P$  over  $R$ .
23. Given  $P = 3x - 2y$ , find the minimum value of  $P$  over  $R$ .

Refer to the figure below for problems 24 and 25.

$$\begin{aligned} 2x + y &\leq 10 \\ 5x + y &\geq 15 \\ x, y &\geq 0 \end{aligned}$$



24. Which of regions  $A, B, C$ , or  $D$  is the feasible region for the system of inequalities?

25. Find the corner points of the feasible region.

APPLICATIONS (10 POINTS EACH, PARTIAL CREDIT)

26. Consider the function

$$f(x) = \frac{2x}{(x+3)(x-2)}.$$

(a) What is the domain of  $f(x)$ ?

(b) Evaluate  $f(2)$ .

(c) Find any horizontal asymptotes of  $f(x)$ .

(d) Find any vertical asymptotes of  $f(x)$ .

27. World population, in billions of people, can be modelled with the equation  $N(t) = N_0 e^{0.0114t}$ , where  $t$  is the number of years since 1995. Assume the population in 1995 was 5.7 billion.

(a) According to this model, what will the world population be in the year 2005?

(b) How many years (after 1995) will it take for the population to reach 8 billion?

28. Suppose you purchase a new home for \$280,000. The mortgage company agrees to lend you that amount at an annual interest rate of 6.5% compounded monthly.

(a) How much are your monthly payments for 30-year loan?

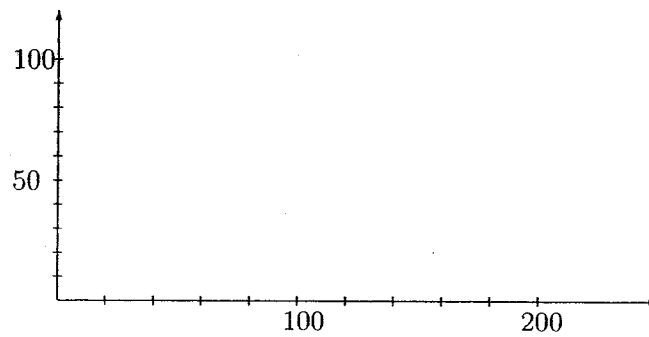
(b) Compute the total interest that you pay to the mortgage company with the 30-year loan.

(c) How much are your monthly payments if you choose a 20-year loan?

29. A vineyard produces two special wines, a white and a red. A bottle of the white wine requires 4 pounds of grapes and 1 hour of processing time. A bottle of the red wine requires 5 pounds of grapes and 2 hours of processing time. The vineyard has on hand 420 pounds of grapes and can allot 150 hours of processing time to the production of these wines. The profit on a bottle of the white wine is \$11.00, while the profit on a bottle of the red wine is \$20.00. The goal is to determine the number of bottles of red and white wine that should be produced in order to maximize the profit.

(a) Set up the linear programming problem that needs to be solved to answer the question.

(b) Graph the feasible region for this problem.



(c) Give the coordinates of the corner points of the feasible region.

(d) Find the number of bottles of red wine and the number of bottles of white wine needed to maximize the profit. What is the maximum profit?

30. Given a standard 52-card deck, answer the following questions.

(a) How many different 5-card hands are there?

(b) How many 5-card hands contain exactly 3 kings?

(c) What is the probability of being dealt a 5-card hand with exactly 3 kings?

(d) What is the probability of being dealt a 5-card hand with at least one king?

## Facts and Formulas

### Algebra

$$a^x \cdot a^y = a^{x+y} \quad \frac{a^x}{a^y} = a^{x-y} \quad (a^x)^y = a^{xy} \quad \left(\frac{a}{b}\right)^x = \frac{a^x}{b^x} \quad (ab)^x = a^x b^x$$

$$a^x = a^y \text{ iff } x = y \quad a^x = b^x \text{ iff } a = b, x \neq 0$$

$$\log_b 1 = 0 \quad \log_b b = 1 \quad \log_b b^x = x \quad b^{\log_b x} = x \quad (x > 0)$$

$$\log_b(xy) = \log_b x + \log_b y \quad \log_b(x^n) = n \log_b x \quad \log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y \quad \log_b x = \log_b y \text{ iff } x = y$$

$$\text{Cost function: } C = (\text{fixed costs}) + (\text{variable costs}) = a + bx$$

$$\text{Price-demand: } p = m - nx$$

$$\text{Revenue function: } R = xp = x(m - nx)$$

$$\text{Profit function: } P = R - C = x(m - nx) - (a + bx)$$

### Finance

$$\text{Simple interest: } I = Prt$$

$$\text{Simple interest: } A = P(1 + rt)$$

$$\text{Compound interest: } A = P\left(1 + \frac{r}{m}\right)^n \text{ or } A = P\left(1 + \frac{r}{m}\right)^{mt}$$

$$\text{Continuous interest: } A = Pe^{rt}$$

$$\text{Annual percentage yield: } APY = \left(1 + \frac{r}{m}\right)^m - 1$$

$$\text{Future value ordinary annuity: } FV = PMT \frac{(1+i)^n - 1}{i}$$

$$\text{Present value ordinary annuity: } PV = PMT \frac{1 - (1+i)^{-n}}{i}$$

### Counting and Probability

$$\text{Permutations: } {}_n P_k = \frac{n!}{(n-k)!}$$

$$\text{Combinations: } {}_n C_k = \frac{n!}{(n-k)! k!}$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$