

# MATH 1070 UNIFORM FINAL EXAM

December 7, 2002

Print your name \_\_\_\_\_

Circle your section:

Byrne	Hebert	Hebert	Landry	McGowan	Sullivan	Yoo
Online	MW 11:30	MW 1:00	TTh 8:30	TTh 10:00	TTh 5:30	TTh 2:30

**Instructions:** Read the problems carefully. If you do not understand a question, ask for clarification. You have three hours to complete the exam. You may use a calculator and the page of formulas provided with the exam. The maximum score for this exam is 100 points: 50 points for Part 1 and 50 points for Part 2. To receive credit for Part 1 questions, you must transfer your answers to the answer sheet. To receive **partial** credit for Part 2 questions, you must **show all work**. Good luck!

DO NOT WRITE BELOW THIS LINE

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Part 1: Short Answer (50 pts) \_\_\_\_\_

26. (10 pts) \_\_\_\_\_

27. (10 pts) \_\_\_\_\_

28. (10 pts) \_\_\_\_\_

29. (10 pts) \_\_\_\_\_

30. (10 pts) \_\_\_\_\_

Total (100 pts): \_\_\_\_\_

# ANSWER SHEET FOR PART 1

You must transfer your answers to questions in Part 1 to this sheet.

Print your name and section \_\_\_\_\_

- 1. \_\_\_\_\_
- 2. \_\_\_\_\_
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- 21. \_\_\_\_\_
- 22. \_\_\_\_\_
- 23. \_\_\_\_\_
- 24. \_\_\_\_\_
- 25. \_\_\_\_\_

Total points \_\_\_\_\_

Print your name and section \_\_\_\_\_

**SHORT ANSWER (2 POINTS EACH, NO PARTIAL CREDIT, TRANSFER YOUR ANSWERS TO THE ANSWER SHEET)**

Functions

1. Find the equation of the line that passes through the point  $(-4, -6)$  with a slope of 2.
2. Find the domain of the function  $f(x) = x^3 + x^2 - 4x - 1$ .
3. Find the domain of the function  $h(x) = \sqrt{4x - 1}$ .
4. Find the solution of the equation  $\ln x + \ln(x + 6) = \ln 16$ .
5. Find the solution of the equation  $3^{2x} = 9^{3x-1}$ .
6. Use a graphing utility to find the zeros of the function  $f(x) = -x^4 + 3x^3 - 3x^2 + x + 10$  (to two decimal places).

Probability

7. How many three-letter code words can be formed from the first five letters of the alphabet assuming that no letter can be used more than once?
8. Two balls are drawn without replacement from a container with three distinct yellow balls and two distinct red balls. What is the probability that two yellow balls are drawn?
9. A class has 8 boys and 12 girls. How many different ways can 1 boy and 2 girls be selected as representatives to student government?
10. Find the value of  $x$ ,  $y$ , and  $z$  in the following table given that  $n(A) = 80$ ,  $n(B) = 70$ , and  $n(A \cup B) = 110$ .

	A	A'	Total
B	x	y	
B'	z		
Total			200

11. Using the table in the previous problem, find  $n((A \cup B)')$ .

Finance

12. Suppose you invest \$10,000 in an account that earns interest at a rate of 7% compounded monthly. How much money is in the account after 10 years?
13. Suppose you invest \$10,000 in an account that earns interest at a rate of 6% compounded continuously. How much money is in the account after 10 years?

14. How long will it take \$2000 to grow to \$10,000 if placed in an account that compounds continuously at a rate of 6.5% per year?
15. What is the future value of an annuity in which payments of \$350 are made monthly and interest is compounded monthly at an APR of 4%?
16. How much money should be invested in an account with an APR of 6.23% compounded annually in order to accumulate \$100,000 in ten years?

Linear Systems and Inequalities

17. Solve the following system of equations using any method.

$$\begin{aligned} 3x - y &= -5 \\ 4x - 4y &= -12 \end{aligned}$$

18. Solve the following system of equations using any method.

$$\begin{aligned} 0.4x + 0.7y &= -1 \\ -0.8x - 1.4y &= 0 \end{aligned}$$

19. Find  $4BA$  given that

$$A = \begin{pmatrix} 4 \\ -3 \\ 6 \end{pmatrix} \quad \text{and} \quad B = (a \ b \ c)$$

20. Evaluate

$$\begin{pmatrix} -1 & 5 & 4 \\ 3 & 2 & -2 \end{pmatrix} \begin{pmatrix} -2 & 0 \\ 1 & 3 \\ 0 & -3 \end{pmatrix}$$

21. Find  $x$  and  $y$  so that

$$\begin{pmatrix} x & 2 \\ -1 & 2x \end{pmatrix} + \begin{pmatrix} y & 3 \\ 0 & y \end{pmatrix} = \begin{pmatrix} -1 & 5 \\ -1 & 1 \end{pmatrix}.$$

22. What is the inverse of the following matrix?

$$\begin{pmatrix} -1 & -2 & 2 \\ 4 & 3 & 0 \\ 4 & 0 & 4 \end{pmatrix}.$$

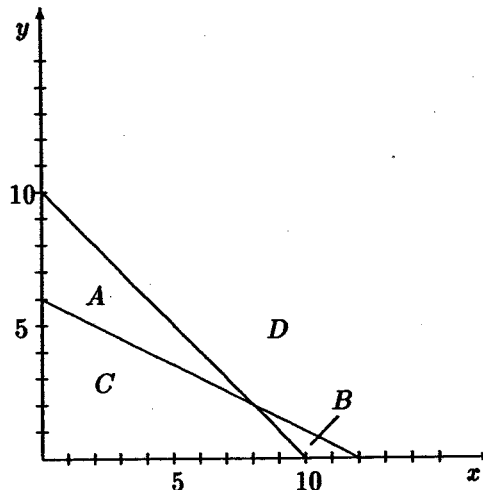
23. Consider the following system of inequalities.

$$\begin{aligned} 4x + y &\leq 16 \\ 3x + y &\leq 14 \\ x, y &\geq 0 \end{aligned}$$

The corner points for the bounded feasible region are  $O = (0, 0)$ ,  $A = (0, 14)$ ,  $B = (2, 8)$  and  $C = (4, 0)$ . Find the maximum value of the objective function  $P = 3x + 2y$ .

Refer to the feasible region and graph below for problems 24 and 25.

$$\begin{aligned} x + y &\leq 10 \\ x + 2y &\leq 12 \\ x, y &\geq 0 \end{aligned}$$



24. Which of regions  $A, B, C$ , or  $D$  is the feasible region for the system of inequalities?

25. Find the point *in the first quadrant* at which the two boundary lines intersect.

APPLICATIONS (10 POINTS EACH, PARTIAL CREDIT)

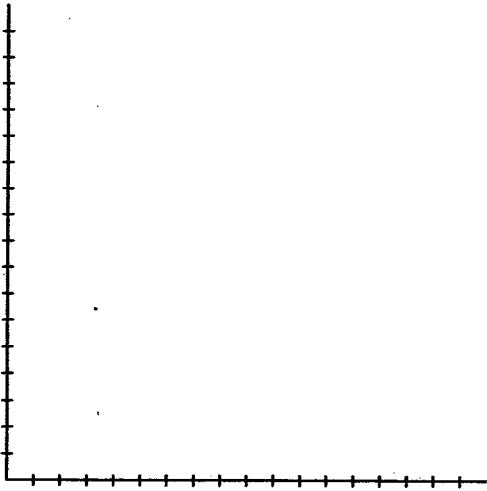
26. The cost of manufacturing 2500 radios is \$18,000 and the cost of manufacturing 3300 radios is \$22,000.

(a) Find the linear cost function  $C(x)$  for manufacturing  $x$  radios.

(b) What is the cost of manufacturing 5281 radios?

(c) What does the  $y$ -intercept of the cost equation represent?

- (d) The average cost function is defined to be  $\bar{C}(x) = \frac{C(x)}{x}$ . Find the average cost function and graph it, labeling the axes appropriately.



27. A software company plans to build a new manufacturing center in the next three to five years. The cost of the building plus start-up costs is \$15 million.
- (a) Suppose the company decides to make a single deposit in an account with continuous compounding at 8.00% per year. How much should be invested to cover the costs of the new center if it is to be built five years in the future?

- (b) Alternatively, suppose the company chooses to invest funds quarterly in an account with quarterly compounding at an annual rate of 7.00%. Under this plan, how much should be invested each quarter if the center is to be built in three years (12 equal payments)?

28. A baker makes three emergency grocery trips for milk, eggs, and sugar. The first trip was to purchase 1 gallon of milk, 1 pound of sugar, and 3 dozen eggs. The total spent was \$7.50. The second trip was to purchase 2 gallons of milk, 4 pounds of sugar, and 2 dozen eggs. The total spent was \$14.00. The third trip was to purchase 2 gallons of milk, 3 pounds of sugar, and 3 dozen eggs. The total spent was \$13.50.

- (a) If  $x_1, x_2, x_3$  are used to represent the quantities of milk (in gallons), sugar (in pounds), and eggs (in dozens), respectively, write a system of linear equations describing the purchases on the three trips to the store.

- (b) Write the above system of equations as a matrix equation of the form  $Ax = b$ .

- (c) What is the inverse of the matrix  $A$ ?

(d) Solve the system of equations by solving the matrix equation  $x = A^{-1}b$ .

29. A small accounting firm prepares tax returns for two types of customers: individuals and small businesses. A tax return is prepared in two steps: data from an interview with the client are entered into a computer system and the computer prepares the tax return. Data entry for an individual requires 2.5 hours; data entry for a small business requires 3.0 hours. The computer requires 20 minutes to prepare an individual tax return and 30 minutes to prepare a small business tax return. A total of 120 hours per week are available for data entry and the computer is available for 15 hours per week. The firm makes a profit of \$100 on each individual tax return and \$175 on each small business tax return.

(a) Formulate this problem as a linear program.

(b) Graph the feasible region and find the corner points.

(c) How many of each type of tax return should the firm schedule each week to maximize its profits?

30. Suppose that 6 women and 7 men are candidates for 5 positions. Assume that the 5 positions are filled randomly using the 13 candidates.

(a) How many ways can the 5 positions be filled?

(b) How many ways can the 5 positions be filled with 2 women and 3 men?

(c) What is the probability that the 5 positions will be filled with 2 women and 3 men?

(d) What is the probability that at least three women will be chosen for the 5 positions?

## Facts and Formulas

### Algebra

$$a^x \cdot a^y = a^{x+y} \quad \frac{a^x}{a^y} = a^{x-y} \quad (a^x)^y = a^{xy} \quad \left(\frac{a}{b}\right)^x = \frac{a^x}{b^x} \quad (ab)^x = a^x b^x$$

$$a^x = a^y \text{ iff } x = y \quad a^x = b^x \text{ iff } a = b, x \neq 0$$

$$\log_b 1 = 0 \quad \log_b b = 1 \quad \log_b b^x = x \quad b^{\log_b x} = x \quad (x > 0)$$

$$\log_b(xy) = \log_b x + \log_b y \quad \log_b(x^n) = n \log_b x \quad \log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y \quad \log_b x = \log_b y \text{ iff } x = y$$

$$\text{Cost function: } C = (\text{fixed costs}) + (\text{variable costs}) = a + bx$$

$$\text{Price-demand: } p = m - nx$$

$$\text{Revenue function: } R = xp = x(m - nx)$$

$$\text{Profit function: } P = R - C = x(m - nx) - (a + bx)$$

### Finance

$$\text{Simple interest: } I = Prt$$

$$\text{Simple interest: } A = P(1 + rt)$$

$$\text{Compound interest: } A = P\left(1 + \frac{r}{m}\right)^n \text{ or } A = P\left(1 + \frac{r}{m}\right)^{mt}$$

$$\text{Continuous interest: } A = Pe^{rt}$$

$$\text{Annual percentage yield: } APY = \left(1 + \frac{r}{m}\right)^m - 1$$

$$\text{Future value ordinary annuity: } FV = PMT \frac{(1+i)^n - 1}{i}$$

$$\text{Present value ordinary annuity: } PV = PMT \frac{1 - (1+i)^{-n}}{i}$$

### Counting and Probability

$$\text{Permutations: } {}_n P_k = \frac{n!}{(n-k)!}$$

$$\text{Combinations: } {}_n C_k = \frac{n!}{(n-k)! k!}$$

$$P(A \cap B) = P(A) \cdot P(B) \quad (A \text{ and } B \text{ independent})$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A') = 1 - P(A)$$