ITERATIVE GAUSS-NEWTON-TYPE ALGORITHM FOR NONLINEAR UNSTABLE OPERATOR EQUATIONS WITH SIMULTANEOUS UPDATES OF THE REGULARIZED FRÉCHET DERIVATIVE

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Consider a nonlinear operator equation in the form:

$$F(x) = 0, \quad F : H_1 \to H_2, \tag{1}$$

on a pair of Hilbert spaces H_1 and H_2. Assume that equation (1) has a solution $\hat{x}$, not necessarily unique, and the operator F is twice Fréchet differentiable without such structural assumptions as monotonicity, invertibility of $F'(x)$ etc.

To avoid the ill-posed inversion of the Fréchet derivative operator $F'(x)$ various discrete and continuous methods based on a regularization were suggested. A principal point in the numerical implementation of regularized Newton's and Gauss-Newton's procedures is the computation of the operators $(F'(x) + \varepsilon I)^{-1}$ and $(F'^*(x)F'(x) + \varepsilon I)^{-1}$ respectively. This computation for certain operators requires a considerable effort in many applications. Besides it may decrease the accuracy of the approximate solution. In order to deal with it, a novel iteratively regularized algorithm with simultaneous updates of the operator $(F'^*(x_n)F'(x_n) + \varepsilon_n I)^{-1}$ is proposed:

$$x_{n+1} = x_n - B_n[F'^*(x_n)F(x_n) + \varepsilon_n(x_n - x_0)], \tag{2}$$

$$B_{n+1} = [I - \lambda(F'^*(x_n)F'(x_n) + \varepsilon_n I)]B_n + \lambda I, \tag{3}$$

$$x_0 \in H_1, \quad B_0 \in L(H_1), \quad 0 < \varepsilon_n \to 0 \quad \text{as} \quad n \to \infty, \quad \lambda > 0.$$

A convergence theorem is proved. The stability of the process towards noise in the data is analyzed, and a stopping time is chosen so that the method converges as the noise level tends to zero. The scheme is illustrated by a numerical example in which a nonlinear inverse problem of gravitational sounding is investigated. Based on theoretical and numerical results the recommendations on the choice of $\{\varepsilon_n\}$, λ and B_0 are given.

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