EFFICIENT AND NOVEL ITERATIVE ALGORITHMS FOR SOLUTION OF REGULARIZED TOTAL LEAST SQUARES

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Abstract. Error-contaminated systems $Ax \approx b$, for which A is ill-conditioned, are considered. Such systems may be solved using Tikhonov-like regularized total least squares (RTLS) methods. Golub, Hansen and O'Leary, 1999, presented a parameter dependent direct algorithm for the solution of the augmented Lagrange formulation for the RTLS problem. Guo and Renaut, 2001 derived an eigenproblem for the RTLS which can be solved using the iterative inverse power method provided a physical constraint parameter is known. Here we present an alternative derivation of the eigenproblem for constrained TLS through the augmented Lagrangian for the constrained normalized residual. This extends the analysis of the eigenproblem and leads to derivation of more efficient algorithms.

Guo and Renaut, [1], obtained the solution of the RTLS problem by finding the minimum eigenpair for an augmented solution-dependent block matrix. The eigenpair is found iteratively, using inverse iteration applied to the solution dependent matrix. An efficient solution technique based on block Gaussian elimination and the generalized singular value decomposition(GSVD), [2], is presented. For cases of high noise a bisection search strategy assists with enforcing the constraint condition. We also provide an L-curve approach for cases in which a good estimate of the physical constraint parameter is not available. These algorithms vary with respect to the parameters which need to be prescribed. Numerical and thoretical results supporting the different versions will be presented.

Key words. Total least squares, regularization, ill-posedness, Rayleigh quotient iteration

AMS subject classifications. 65F20, 65F30

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