Multilevel preconditioners for solving eigenvalue problems occuring in the design of resonant cavities

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Abstract

We investigate eigensolvers for computing a few of the smallest eigenvalues of a constrained generalized eigenvalue problem

$$A\mathbf{x} = \lambda M\mathbf{x}, \qquad C^T\mathbf{x} = \mathbf{0},$$

resulting from the finite element discretization of the time independent Maxwell equation. We found the Jacobi-Davidson algorithm (JD) and the locally optimal block preconditioned conjugate gradient (LOBPCG) method to be the most effective factorization-free algorithms for solving this eigenvalue problem provided that they are complemented by a powerful preconditioner. We compare various combinations of hierarchical basis and AMG preconditioners that improve the convergence rate of the eigensolvers without consuming memory space excessively. We present numerical results of very large eigenvalue problems originating from the design of resonant cavities of particle accelerators that are not reasonably solvable without the above techniques.

Key words: Maxwell equation, generalized eigenvalue problem, Jacobi-Davidson, LOBPCG, AMG preconditioner
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