Application of Algebraic Multigrid to Bone Modeling Problems

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Trabecular bone is the primary load bearing biological tissue in the spine and in the hip, and has a complex structure with over 80% porosity. A common method to determine the structural properties of trabecular bone is to use 3D digital images obtained from micro computed tomography to construct a finite element (FE) mesh of voxel elements. These voxel meshes have the advantage of being able to capture complex geometries but require many elements to accurately model the mechanics [4]. For example, a convergent linear analysis of one human vertebral body requires over 44 million elements. Smoothed aggregation is an algebraic multigrid algorithm that has been shown to be effective in elasticity problems [5, 2]. This study will discuss some of the first applications of highly scalable linear solver algorithms to whole vertebral body high-resolution structural analysis.

We use a custom code *Bobcat* to translate the digital images obtained from micro computed tomography, our parallel FE code *Athena* (built on a serial research FE code *FEAP*) and our implementation of a smoothed aggregation multigrid method in the linear solver package *Prometheus*. Prometheus is built on *PETSc* from Argonne National Laboratory and *ParMetis* from University of Minnesota. Athena has been designed to take advantage of the cluster of symmetric multiprocessor (SMP) architecture of the IBM SP by partitioning first to the nodes and then, on each node, to the processors to take advantage of faster communication within a node, though Prometheus, PETSc and ParMETIS are all pure MPI codes. The IBM Power3 which runs at 375 MHz Power3 processors with 4 floating point instructions per cycle for peak flop rate of 1.5 Gflops. We use the IBM SP at Lawrence Livermore National Laboratory (LLNL), with 16 processors per node and the IBM SP at San Diego Super Computing Center with 8 processors per node for this study (available through NPACI).

We conduct a scaled speedup study of one vertebral body model with up to 203 million dof. Coarser versions of the problem were run with a number of nodes selected to keep approximately the same number of equations per nodes as the largest, 203 million dof, problem, run on 64 nodes. We are able to solve the problem in about 3 minutes, after the initial FE paritioning and multigrid setup phase, with about 80% parallel efficiency from 1 to 62 nodes, and about 5% of theoretical peak, on the IBM SP at LLNL with Chebyshev smoothers [3], and symmetric Gauss-Seidel smoothers [1].

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