MATH 3200 **Review for Test 1** Bennethum

These are practice problems for test 1. For this exam, one side of an 8.5x11" sheet of paper will be allowed for notes. No technology of any kind will be allowed. The topics covered on this exam include material covered in Chapters 1 and 2. Long application problems will not be on the exam (this is why you have a project due). Answers are given on the last page.

1. Classify the following ODE. Determine the order, whether it is linear or nonlinear, whether it is autonomous, and if linear, whether it is homogeneous. Circle all that are correct.

   (a) $t^4 y' + y\sin(t) = 6$
   A) first-order, second-order, third-order B) linear, nonlinear
   C) homogeneous, non-homogeneous D) autonomous, non-autonomous

   (b) $yy'' = x^3 + y\sin(3x)$
   A) first-order, second-order, third-order B) linear, nonlinear
   C) homogeneous, non-homogeneous D) autonomous, non-autonomous

   (c) $(y')^3 + t^5 \sin y = y^4$
   A) first-order, second-order, third-order B) linear, nonlinear
   C) homogeneous, non-homogeneous D) autonomous, non-autonomous

   (d) $\dfrac{y'}{(x^2+1)y} = \cos x$
   A) first-order, second-order, third-order B) linear, nonlinear
   C) homogeneous, non-homogeneous D) autonomous, non-autonomous

2. Determine whether the function is a solution to the following particular problem. If not, explain why.

   (a) $y = x^2 \qquad y' = xy^2 - 2x, \qquad y(1) = 1$

   (b) $y = 3\sin 2t + e^{-t} \qquad y'' + 4y = 5e^{-t}, \qquad y(0) = 1$

3. Find the general solution of the following equations. The answer may be given in implicit form, but reasonable simplification is expected. One of the following is not solvable by any method we have discussed. For this problem, just write *not solvable by* and list the techniques you considered.

   (a) $y' = 3y$

   (b) $y' = x^2(1+y)$

   (c) $\dfrac{dy}{dx} = -\dfrac{1 + e^x y + xe^x y}{xe^x + 2}$

   (d) $\dfrac{dy}{dx} = y + e^{3x}$

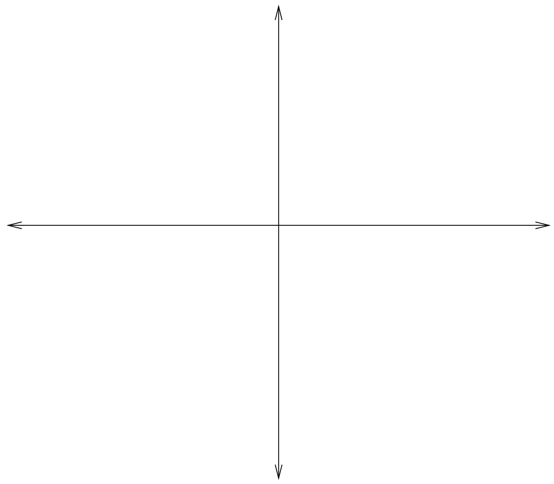
   (e) $(y - x)y' = x + 2y$

4. Find the particular solution of the following equation.

$$y' + 2ty = 2t, \qquad y(0) = 4$$

5. Consider the ODE $\dfrac{dy}{dt} = 2t + y$

   (a) Graph the direction field for the ODE



   (b) Solve the initial value problem with initial condition $y(-1) = 0$.

   (c) Sketch the solution curve that passes through $y(-1) = 0$ on your direction field plot.

   (d) Do your answers make sense? Why or why not?

6. True or False (circle one): Based on existence and uniqueness theorems

   (a) True False

   The solution to $y' = \dfrac{t}{1-y^2}, \qquad y(0) = \dfrac{1}{2}$ is guaranteed to exist and be unique in a region which includes all values of $t$.

   (b) True False

   The solution to $y' + \frac{1}{1-t}y = 0, \qquad y(0) = \frac{1}{2}$ is guaranteed to exist and be unique in a region which includes $-\infty < t < 1$.

7. Use Euler's method to approximate the solution to the initial value problem

$$y' = -y + t \qquad y(0) = 0.$$

   (a) Determine $y_1$ and $y_2$ in terms of an arbitrary $h$.

   (b) Determine $y_1$ and $y_2$ if $h = 0.1$.

8. Suppose a brine (salt-water) containing 3 kg of salt per liter (L) runs into a tank initially filled with 400 L of water containing 20 kg of salt. The brine enters at 10 L/min, the mixture is kept uniform by stirring, and the mixture flows out at the same rate.

(a) Determine the governing ODE.

(b) Solve the ODE and find $Q(t)$ for $t \geq 0$.

(c) Based on your solution, what is $\lim_{t \to \infty} Q(t)$? Is this consistent with the physics of the problem? Explain.

1a) first-order, linear, non-homogeneous, non-autonomous 1b) second-order, non-linear, non-autonomous
1c) first-order, nonlinear, non-autonomous 1d) first-order, linear, homogeneous, non-autonomous
2a) Is not a solution - function does not satisfy ODE. 2b) Is a solution - function satisfies ODE and initial condition
3a) Guess and check: $y = Ce^{3t}$ or $y = Ce^{3x}$ 3b) Separation of variables $y = Ce^{\frac{x^3}{3}} - 1$
3c) Exact differential $xe^x y + 2y + x = C$ 3d) Integrating factor $y = \frac{1}{2}e^{3x} + Ce^x$
3e) Not solvable by: Integrating factor, Exact differential, Separation of Variables, and it's not a Bernoulli differential equation
4) $y = 3e^{-t^2} + 1$.
5a) $dy/dt = 2t + y$ has the following isoclines: slope is zero along the line $y = -2t$, slope is 1 along $y = -2t + 1$, slope is -1 along $y = -2t - 1$ etc. You should draw about 5 isoclines.
5b) Solving using integrating factor (it's a linear equation) we have $y = -2t - t$.
5c) So the solution lies on top of the isocline: $y = -2t - 2$, which also happens to have a slope of $-2$.
5d) The solution makes sense because it is tangent to the slope field. Note that if something isn't consistent, and you recognize this, then you know you have an error and you should state this. Knowing when something isn't right is important!
6a) This is false. No explanation required but here it is: This is a nonlinear ODE so we only know that the solution exists in some sub-region of the region for which the right-hand-side is continuous and its first partial with respect to the dependent variable ($y$ in this case) is continuous. For this problem the region which contains the IC is: $-\infty < t < \infty, \quad -1 < y < 1$.
6b) True. No explanation required but here it is: This is a linear ODE so the solution exists and is unique in the region where $p(t)$ and $g(t)$ (right-hand-side) are continuous. Note that there is a discontinuity at $t = 1$. The IC lies to the right of $t = 1$, so the region guaranteed to be unique and exist is $-\infty < t < 1$.
7a) $y(h) = 0 \quad y(2h) = h^2$
7b) $y(0.1) = 0 \quad y(0.2) = .1^2 = .01$
Remark (not possible to do on an exam, but as a check here): The exact solution is $y = t - 1 + e^{-t}$. Thus the exact values at $h$ and $2h$ are $y(0.1) = 0.004837 \quad y(0.2) = .0187$ which are (relatively) close to the values gotten using Euler's method.
8a) Let $Q(t)$ be the mass of brine in Kg in the tank at time $t$ (in minutes). The ODE is $\frac{dQ}{dt} = 30 - \frac{1}{40}Q$.
8b) Using the IC $Q(0) = 20$ Kg, we get (solving either by Integrating Factor or separation of variables): $Q = -1180e^{-t/40} + 1200$.
8c) $\lim_{t\to\infty} Q(t) = 1200$. This makes sense with the physics. At long time we would expect the concentration of salt in the tank to be equal to the concentration of the brine entering the tank. This would make the total mass of salt in the tank to be 3 (Kg/L) * 400L = 1200 Kg.