

Math 4791/5791 - Problem Set 1
Fall 2008

Due Wed., August 20

Do not wait until the last minute to start the assignment! It's best to get an early start and work on it in several steps. Be sure to present the domain and units where appropriate and write up your solution as if you were presenting it to a client, complete with assumptions stated explicitly.

1. **BB Two population models.** Consider the two following population models.
 - (a) **BB A** (fictitious) wildlife study of elk in Rocky Mountain National Park was conducted in 1990. At that time it was determined that there were 4500 elk in the Park and, based on data from previous years, the population was growing at a rate of 300 elk per year. Assuming that growth rate remains constant, find the elk population in 1995 and 2000. Determine the function that gives the elk population for all times after 1990.
 - (b) **BB As** you may know the world population reached 5 billion in 1988 and was growing at a rate of 1.6% per year. Assuming that growth rate remains constant, determine the function that describes the world population growth for all times after 1988. According to this function, when will the population reach 6 billion? According to this function, how many people were added (net gain) in 1990? How many people will be added (net gain) in 2000? What is the doubling time of the world's population assuming that the growth rate remains constant? *If you assume that the human race began with two individuals, what date does this model give for the "creation"? Is the model accurate? Why or why not?*
 - (c) Carefully contrast the mathematics of these two population models.
2. **BB Prairie dogs.** A colony of prairie dogs increases in size at a constant rate of 15% per year starting with a known initial population of P_0 . Find two equivalent functions that give the prairie dog population at all times; one function should be base e and the other function may have any other base. How long does it take the colony to double in size? to triple in size? to increase tenfold in size?
3. **BB Mixing tank.** A 1000-liter mixing tank filled with brine is fed from a supply tank by an inflow pipe at a rate of 10 liters per minute and drained by an outflow pipe at the same rate. The initial brine concentration in the mixing tank is 50 grams per liter and the concentration of the solution in the supply tank is 300 grams per liter. Set up the ODE, determine the coefficients, and then find the function that gives the brine concentration in the mixing tank at all times. Analyze, graph, and discuss your solution.
4. **BB Variable volume mixing tank.** (Required of graduate students; recommended for undergraduates). Consider the mixing tank of the previous problem, but now assume that the mixing tank has a capacity of 2000 liters. Assume also that the mixing tank initially contains 1000 liters of brine with a concentration of 50 grams per liter. Finally assume that the inflow rate to the tank is 10 liters per minute and the outflow rate is 8 liters per minute. Find the function that gives the brine concentration in the mixing tank at all times. When does the experiment "end"? Analyze, graph, and discuss your solution.